

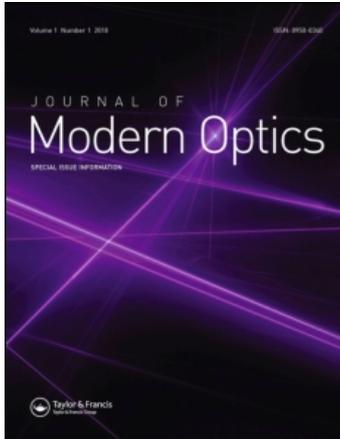
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## Journal of Modern Optics

Publication details, including instructions for authors and subscription information:

<http://www.informaworld.com/smpp/title~content=t713191304>

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**To cite this Article** Torres-Cisneros, G. E. , Romero-Troncoso, R. J. , Sánchez-Mondragón, J. J. and Alvarado-Méndez, E. (1995) 'Collision of Two Real Dark Spatial Solitons', Journal of Modern Optics, 42: 11, 2323 – 2328

**To link to this Article:** DOI: 10.1080/09500349514552011

**URL:** <http://dx.doi.org/10.1080/09500349514552011>

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## Collision of two real dark spatial solitons

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*(Received 10 October 1994; revision received 20 March 1995)*

**Abstract.** We study the collision of two dark spatial solitons immersed in their own finite background beams. We show that real spatial solitons can cross each other only for certain combinations of the collision angle and the intensities of the beams. Otherwise, the collision pattern evolves into an array of quasi-dark solitons due to the internal gain caused by the modulation instability effect.

### 1. Introduction

Shortly after the first experimental observations of temporal dark solitons in optical fibres based on specialized pulse-shaping techniques [1, 2], a simpler alternative based on the collision of two pulses was experimentally found [3, 4]. There, it was reported that the nonlinear temporal interaction of two visible optical pulses co-propagating in the normal dispersion regime of an optical fibre leads to the formation of a train of dark solitons when the initial separation between them is small enough. This is an interesting temporal result, but its spatial analogy is of remarkable current interest in connection with optical waveguides based on dark spatial solitons (DSSs). Therein lies the interest to carry out a thorough theoretical analysis motivated by the temporal experimental evidence to carry out an analogous DSS experiment.

There are several successful techniques used to generate DSSs. Among them, the use of amplitude or phase masks to produce an adequate initial condition [5], and the adiabatic amplification of a sinusoidal input signal, recently demonstrated in an elegant experiment [6]. The last was inspired by numerical predictions concerning the nonlinear competition between the positive group velocity dispersion effect, the intensity-dependent refractive index of the fibre and the presence of an external gain mechanism leading to the generation of a train of temporal dark solitons [7].

In this paper we consider conditions closer to experimental reality and study the collision of DSSs supported by their own light beams. These are the so-called real

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dark spatial solitons (RDSSs). We show, in the spatial case, that indeed there is a physical connection between dark soliton generation by collision of two close temporal pulses [3] and dark soliton generation by amplification of a sinusoidal input signal [7]. In particular, we use the collision of two optical waveguides based on RDSSs; that is, the collision of two dark spatial solitons each one immersed in its own finite background. We also show that for small collision angles the oscillatory collision pattern causes the onset of the modulation instability effect. That is, the internal gain mechanism under which an array of quasi-dark spatial solitons is formed confirming the temporal analogous experiment.

The numerical experiment corresponds to the collision of two RDSSs within a negative Kerr-type medium. We use the usual two-dimensional (2D) approach, in which the evolution of the transverse envelope of a laser beam is governed by the well-known NLSE [8]

$$i\partial A/\partial Z = (1/2)\partial^2 A/\partial X^2 - |A|^2 A, \quad (1)$$

where  $A(X, Z) = E(X, Z)/\sqrt{P_0}$  is the normalized transverse envelope of the electric field,  $P_0$  is the peak intensity of the beam,  $X = x \cos \theta/x_0$ , with  $x_0$  as the beam width,  $Z = z/L_D$ , with  $L_D = n_0 k_0 x_0^2 \cos \theta$ ,  $n_0$  and  $k_0$  are, respectively, the linear diffractive index and the wavenumber, and  $\theta$  is the angle between the beam and the positive  $z$  axes. In equation (1) the relation  $n_0 k_0 x_0^2 = 1/|n_2|k_0 P_0$  has been assumed, where  $n_2$  is the Kerr refractive index of the medium.

To represent the physical situation of two RDSS merging optical waveguides we use the initial condition

$$A(X, 0) = A_0 \exp[-(X-c)^6/2\sigma^6] \exp(-iVX) \tanh(X-c) \\ + A_0 \exp[-(X+c)^6/2\sigma^6] \exp(iVX) \tanh(X+c), \quad (2)$$

where  $\sigma$  represents the widths of the finite bright backgrounds relative to the widths of fundamental dark solitons.  $V = k_0 n_0 x_0 |v|$  is the normalized transverse velocity of the beams, with  $v = \tan \theta$ , and  $2c$  is the initial separation of the pulses. We show the collision characteristics when the normalized transverse velocity  $V$  of the beams is varied, by solving equations (1) and (2) with standard numerical techniques and displaying a typical numerical output.

The collision of the two RDSSs, for  $V = 6$  and  $V = 0.6$ , are shown in figures 1 (a) and (b), respectively. For the relatively large transverse velocity case, plotted in figure 1 (a), the initial beams are broadened (and chirped) by the negative Kerr-type medium, but the dark solitons maintain their identity after the collision. However, for a smaller value of  $V$ , figure 1 (b) demonstrates that the cross of the initial waveguides results in an array of evolving dark solitons, a peculiar structure identical to that obtained in optical fibres [3]. To ease the graphical comparison, in figure 2 we redraw the last curve of the output profiles of the collision of the two RDSSs displayed in figure 1 (full curves), and compare them, on their own scale, with the corresponding linear interference pattern obtained after propagating separately each RDSS the same distance within the nonlinear medium (broken curves). For large  $V$ , figure 2 (a) shows that both curves are identical, and we may say that the waveguides effectively cross. On the contrary, for small  $V$  the structure of the two curves of figure 2 (b) are quite different, and we can surmise that the merging into additional dark solitons does indeed occur. Our numerical simulations also show that the transition between the results displayed in figure 1 (a) and figure 1 (b) occurs

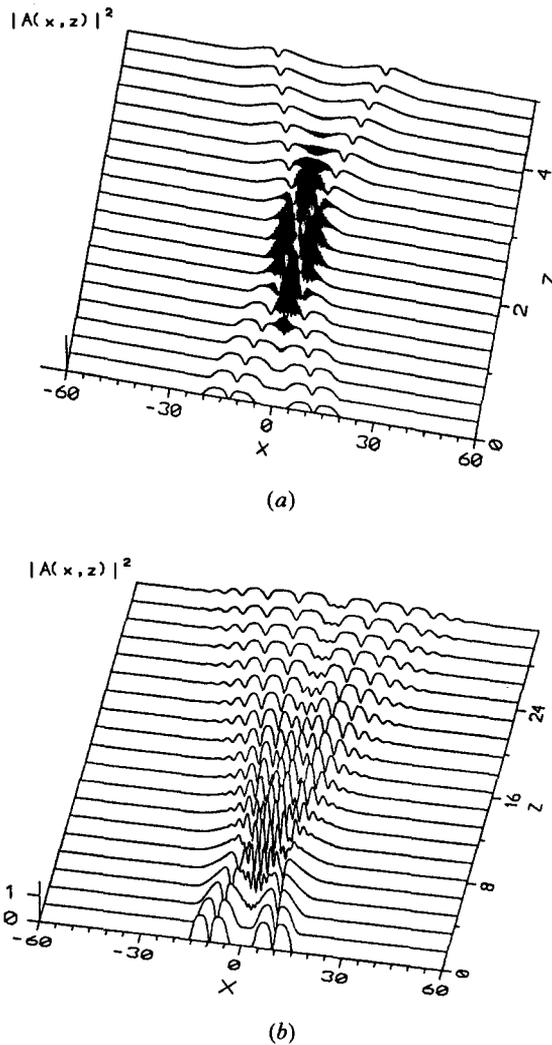


Figure 1. Collision of two real dark spatial solitons for two different transverse velocities. In (a)  $V = 6$  and in (b)  $V = 0.6$ . The other parameters in equation (2) are  $A_0 = 1$ ,  $\sigma = 7$  and  $2c = 25$ .

smoothly as  $V$  decreases. We should stress that the characteristics of a collision of two RDSSs are quite different to those found when the two DSSs are immersed in the same background [9], where an almost ideal behaviour is obtained.

We now give a physical explanation of why two RDSSs give rise to an array of dark fringes for small values of  $V$ . First, we note in figure 1 (b) that the output array of dark fringes separated by a flat bright background steadily grows out of the oscillatory collision profile. Second, the widths of the dark fringes do not change significantly as the collision pattern propagates, in spite of the influence of the self-defocusing medium. These two characteristics resemble those found in the generation of temporal dark solitons by adiabatic amplification of an oscillatory profile [7]. Therefore, the behaviour displayed in figure 1 (b), after the collision of

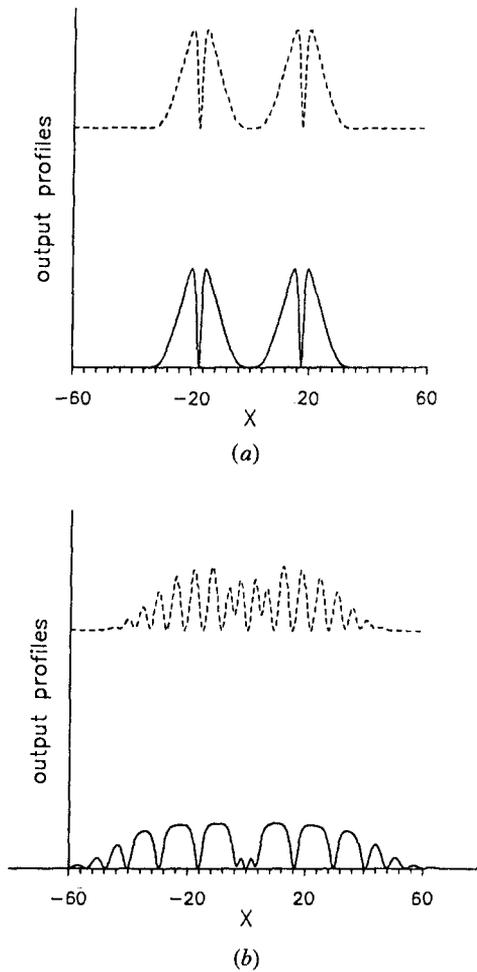


Figure 2. Graphical comparison between the output profiles of the last curves in figure 1 (full curves) and the linear interference patterns of the two corresponding real dark spatial solitons propagated independently (broken curves). The output pulses have been rescaled to facilitate direct comparison.

the two RDSs, can be interpreted as a transient evolution of the collision pattern into an array of dark spatial solitons (immersed into a finite bright background) caused by the presence of an effective gain mechanism.

We claim that such a gain mechanism is provided by the onset of the modulation instability effect caused by cross phase modulation (XPM) between the two RDSSs. A first-order quantitative estimation of this effect can be derived from the following physical model [10]. XPM is implicit in our numerical simulations of equations (1) and (2), but it becomes explicit if we describe the evolution of each RDSS separately by the two coupled equations

$$i\partial A_j/\partial Z = (1/2)\partial^2 A_j/\partial X^2 - (|A_j|^2 + 2|A_{3-j}|^2)A_j \quad \text{for } j = 1, 2. \quad (3)$$

The system of equations (3) accepts the plane-wave solutions  $A_j^{(0)} = A_{j0} \exp(-i\phi_j Z)$ , where  $A_{j0}$  are the initial amplitudes, and  $\phi_j = |A_{j0}|^2 + 2|A_{(3-j)0}|^2$

are the non-linear phases acquired by the beams as they propagate within the Kerr-type medium. Let us assume now that these steady-state solutions are perturbed by periodic (in  $X$ ) waves of small amplitudes of the form  $a_j = \alpha_j \cos(\beta Z - \omega_j X) + i\gamma_j \sin(\beta Z - \omega_j X)$ , where  $\beta$  is the propagation constant and  $\omega_j$  are the frequencies of the perturbative waves. Then, it is possible to show [10] that the steady-state solutions  $A_j^{(0)}$  become unstable for initial amplitudes  $A_{j0}$  and perturbative frequencies  $\omega_j$ , which satisfy the relation

$$\left(\frac{\omega_1^2}{4|A_{10}|^2} + 1\right)\left(\frac{\omega_2^2}{4|A_{20}|^2} + 1\right) < 4. \quad (4)$$

Therefore, the collision of any two beams in a Kerr-type medium, and in particular of two RDSSs, can be viewed as a perturbed signal which will experience gain if the associated perturbative frequencies satisfy equation (4). For a symmetric case, we have  $\omega_1 = \omega_2 = \omega$ ,  $A_{10} = A_{20} = B_0$ , and equation (4) is reduced to  $T > \pi/|B_0|$ , where  $T = 2\pi/\omega$  is the period of the collision pattern and  $B_0$  is its amplitude. Using the linear values for the collision period,  $T = \pi/|V|$ , and for the collision amplitude,  $|B_0| = |A_0|$ , we estimate that the turning of the RDSSs into an array of dark solitons will be evident for  $|V| < |B_0|$ . This is in good quantitative agreement with the results shown in figure 1, where  $A_0 = 1$ . In fact, figure 1(b) shows the possibility of experimental observation of the DSS array at extremely small angles. Notice the relaxation of the usually restrictive propagation requirements in Kerr-type media of just a few diffraction lengths  $L_D$  [6, 11].

In conclusion, we have demonstrated that it is possible to activate the modulation instability effect during a collision of two optical waveguides based on RDSSs, by a proper choice of their intensities and their transverse velocities. The modulation instability acts as an internal mechanism which contributes to the generation of additional dark spatial solitons, as expected from previous experiments [6] and numerical simulations [7] carried out with effective external gain mechanisms. These results also give physical support to the analogous temporal experiment [3]. From the practical point of view, this phenomenon may be applied as an alternative to external-amplification free generation of quasi-dark solitons and as a way to produce an interference pattern with an intensity dependence separation. It also sets the conditions for (controllable) multiple exits, optical junction based on DSSs for weak beams, in a similar manner to those reported with bright spatial solitons [12, 13].

### Acknowledgments

We thank Professor V. A. Vysloukh for his valuable comments. This work was partially supported by the Consejo Nacional de Ciencia y Tecnologia under grant F388-E.

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