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Internal reflection of one-dimensional bright spatial solitons

E. ALVARADO-MÉNDEZ, G. E. TORRES-CISNEROS,
M. TORRES-CISNEROS
Departamento de Electrónica, FIMEE, Universidad de Guanajuato Apdo. Postal 215-A, Salamanca, GTO., 36730 México
J. J. SÁNCHEZ-MONDRAGÓN, V. VYSLOUKH
Instituto Nacional de Astrofísica, Optica y Electrónica; Apdo. Postal 51, Puebla, Pue., 72000 México

We study the reflection of one-dimensional spatial solitons at the non-linear interface between a Kerr-type medium and a linear medium. Our study places emphasis on determining the physical conditions under which the beam reflected by the non-linear interface is still a spatial soliton. We find that for small angles of incidence an elastic internal reflection takes place, in the sense that the reflected soliton is essentially the same as the incident one. For incidence angles near a critical angle, the reflected soliton becomes less intense and its reflection angle is smaller than the angle of incidence. Finally, for spatial solitons with input angles well above the critical angle, the main part of the energy is transmitted to the linear medium, and no soliton is internally reflected.

1. Introduction

The behaviour of electromagnetic radiation falling upon the boundary of two different media has always been an attractive problem from the physical and practical point of view. In particular, the special cases of non-linear optical interfaces, that is, when at least one of the media exhibits some kind of optical non-linearity, have given rise to a broad variety of interesting phenomena. Among them, we can find filamentation [1], optical bistability [2], surface waves [3, 4], etc. Furthermore, the proposed practical applications of these phenomena include controllable scanning beams [5], optical logic gates [6], weak beam amplification [7] and so on.

From the physical point of view, the exact way in which these phenomena occur depends on the specific non-linear media at both sides of the interface. A major emphasis has been given to non-linear interfaces with Kerr-type materials, which include the linear–non-linear [1, 6] and the non-linear–non-linear [8] interfaces, but diffusive [4, 5, 9], saturable and quadratic [3] non-linearities also have been considered.

In the specific case of an optical beam falling upon a linear–Kerr-like interface, previous studies have shown the excitation of spatial solitons by a transmitted beam [1] and of non-linear surface waves [10, 11]. Reflection and transmission of self-focused channels at Kerr-type–Kerr-type and at Kerr-type–saturable absorber interfaces also have been analysed [12, 13]. However, to the best of our knowledge, the particular problem of the internal

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reflection of a bright spatial soliton has not been analysed. This problem may find practical applications when the spatial soliton acts as an optical waveguide for a weak beam of different wavelength [14].

In this paper we analyse, from the numerical and the theoretical point of view, the behavior of a one-dimensional bright spatial soliton in a medium with cubic non-linearity as it reaches the boundary with a linear medium. In particular, we are interested in determining the conditions under which a spatial soliton is obtained after reflection by the interface. As we will see later, a spatial soliton, almost identical to the incident one, is reflected for incidence angles below a critical angle. For intermediate angles of incidence a soliton is also reflected, but it is different from the incident one. A reflected soliton is not obtained for angles of incidence well above the critical one.

In Section 2 we present the physical and the mathematical description of our problem. We define the fundamental physical parameters and we give the modified NLSE which describes the incidence of a bright spatial soliton at the interface with a linear medium. In Section 3, we present the numerical results that characterize our non-linear interface. In particular, we use a numerical algorithm based on the Inverse Scattering Transform (IST) [15] to determine whether the reflected beam is or is not an exact spatial soliton. In Section 4 we apply the particle-like technique [5, 16] to describe the internal reflection of the incident spatial soliton. Finally, Section 5 gives the conclusion of our work.

2. The physical model for the non-linear interface

Figure 1 illustrates the geometry of the non-linear interface in which we are interested. The interface lies in the y–z plane at $x = 0$, it separates a Kerr-like non-linear medium on the

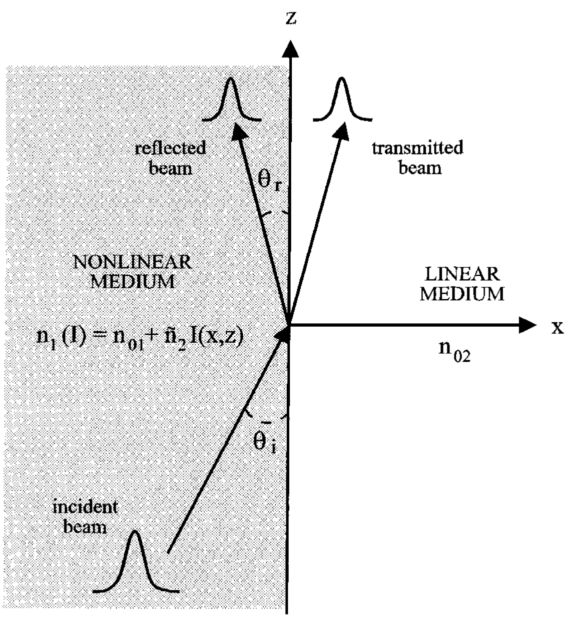

Figure 1 A bright spatial soliton falling upon an interface between a Kerr-like non-linear and a linear media.

left and a linear medium on the right. The non-linear medium is characterized by a refractive index of the form

$$n_1(I) = n_{01} + \tilde{n}_2 I(x,z) \tag{1}$$

where n_{01} is the linear refractive index, $\tilde{n}_2$ is the non-linear refractive index and $I(x,z)$ is the intensity of light. Strictly speaking, Equation 1 describes a Kerr non-linear medium, but we point out that it also may represent some other physical situations of current interest. A photorefractive crystal, for example, exhibits a saturable Kerr non-linearity if the drift mechanism dominates [17, 18]. Therefore, Equation 1 can be valid for experiments with these crystals if the saturation parameter is small enough.

On the other hand, the linear medium on the right of the interface is characterized by a constant refractive index n_{02} and, in order to be able to study the internal reflection of a spatial soliton, it is assumed that $n_{01} > n_{02}$. As it is depicted in Fig. 1, a bright spatial soliton propagating within the non-linear medium falls at an angle θ_i on the interface. Part of its energy is transmitted to the linear medium in the form of a linear beam which will be diffracted, and the remaining energy will be reflected back into the non-linear medium at an angle θ_r. Our problem consists in determining whether this reflected beam is or is not a spatial soliton. In the case that the non-linear interface reflects a soliton, the question is how to obtain its properties, such as its width, its reflection angle, and its transversal spatial shift.

For the mathematical description of our problem we consider an electric field propagating in the positive direction of the z-axis, with a linear polarization parallel to the interface:

$$E_y = \frac{1}{2}\, a(x,z)\, \exp[(\mathrm{i}(\omega t - k_2 z)] \tag{2}$$

where $a(x,z)$ is the transverse beam envelope, ω is the carrier frequency, and k_2 is the wavenumber. We assume that the paraxial approximation is valid and, in consequence, the beam envelope satisfies the equation

$$2\mathrm{i}k_2 \frac{\partial a}{\partial z} = \frac{1}{2}\frac{\partial^2 a}{\partial x^2} + k_2^2 \frac{\delta n(x)}{n_{02}} a \tag{3}$$

where $\delta n(x)$ represents the contribution to the non-linear refractive index. Given the optical properties of the two media at both sides of the non-linear interface, Fig. 1, the refractive index profile takes the form

$$\delta n(x) = [(n_{01} - n_{02}) + \tilde{n}_2 I] f(x) \tag{4}$$

where $f(x)$ is a function which describes the spatial behaviour of the interface. If the interface is abrupt, we can use a step function for $f(x)$; it means $f(x) = U(x)$, where $U(x) = 1$ if $x \leq 0$ and $U(x) = 0$ for $x > 0$. On the other hand, we will use $f(x) = s(x) = 1/2[1 - \tanh(\kappa x)]$ to describe a general interface, where κ represents the physical *steepness* of the interface. Notice that $s(x) \to U(x)$ as κ increases.

If we sustitute Equation 4 into Equation 3 we obtain

$$\mathrm{i}\frac{\partial a}{\partial Z} = \frac{1}{2}\frac{\partial^2 a}{\partial X^2} + Rf(X)[\Delta + |a|^2]a \tag{5}$$

where $\Delta = (n_{01} - n_{02})/\tilde{n}_2|a_0|^2$ represents the normalized refractive index difference; $R = L_\mathrm{d}/L_\mathrm{nl}$, where $L_\mathrm{d} = k_2 n_{02} x_0^2$ is the diffraction length and $L_\mathrm{nl} = (1/k_2\tilde{n}_2|a_0|^2)$ is the characteristic non-linear length. Moreover, $Z = z/L_d$, $X = x/x_0$ and the beam envelope has been normalized to a_0. Finally, x_0 and a_0 are the width and amplitude, respectively, of the initial beam.

Throughout this paper we will concentrate on the internal reflection of first-order bright spatial solitons, and therefore we further assume that x_0 and a_0 satisfy the relation $k_2 n_{02} x_0^2 \tilde{n}_2 |a_0|^2 = 1$, which implies $R = 1$ in Equation 4. Notice that under this assumption

$$a_s(X, Z) = \sec\mathrm{h}[(X - X_0) - VZ]\ \exp[-\mathrm{i}\Phi(X, Z)] \tag{6}$$

is an analytical solution of Equation 5 valid for beams well inside the non-linear medium, where $f(X) = 1$. In Equation 6 $\Phi(x, z) = V(X - X_0) + (1 - V^2)Z/2 + \varphi_0$ is the total phase of the soliton with $V = \tan\theta_i$ being the normalized transverse velocity of the spatial soliton, and with $X_0 < 0$ ($|X_0| \gg 1$) and φ_0 being constants. Equation 6 is no longer valid near the non-linear interface, of course, and it is necessary to consider the spatial dependence of $f(X)$ in Equation 5. In the following section we numerically obtain the fundamental properties of the non-linear interface under the incidence of a bright spatial soliton, and in Section 4 we will give the corresponding analytical and physical support.

3. Numerical simulations of the internal reflection of a spatial soliton

Equation 5 has been solved by standard numerical techniques [19]. We neglect any transient effect in the formation of the spatial soliton within the non-linear medium and assume Equation 6, with $Z = 0$ and $\phi_0 = 0$, as the initial condition for the beam. On the other hand, to avoid numerical noise caused by an abrupt interface, we use $f(X) = s(X)$, with $\kappa = 10$. This proved to be a good representation for the non-linear interface.

Figure 2 shows the total internal reflection of the spatial soliton, obtained for an initial transverse velocity of $V_{in} = 0.5$. As we can see, the initial soliton travels first within the non-linear medium, and then it falls upon the interface, where it is reflected by the non-linear medium. Notice that although the spatial soliton narrows as it approaches the interface, the peak intensity and the width of the reflected soliton are practically the same as those of the initial soliton. Therefore, the soliton energy is conserved during the reflection at the non-linear interface. We will refer to this case as an *elastic* internal reflection of the spatial soliton.

The elastic internal reflection of the soliton is progressively lost as we increase the angle of incidence of the spatial soliton, and we obtain what we will call an *inelastic* internal

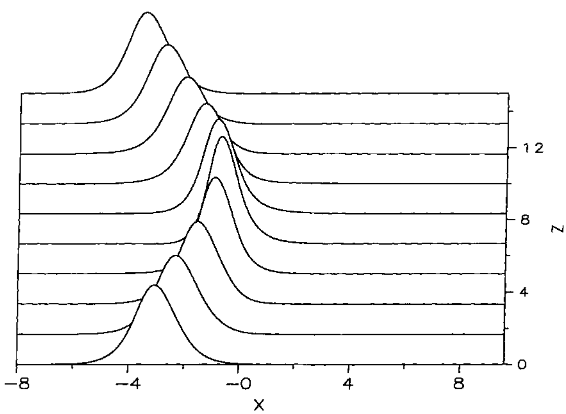

Figure 2 Total internal reflection of a bright spatial soliton obtained for $V = 0.5$. The initial position of the soliton was $x_0 = 3$.

reflection. Figure 3 shows the behaviour of the non-linear interface when the initial transverse velocity of the spatial soliton is (a) $V_{in} = 1.5$ and (b) $V_{in} = 1.9$. In both cases, the soliton clearly penetrates the interface, and part of its initial energy is transmitted to the linear medium as a beam which spreads out due to diffraction. The reflected beams carry only a fraction of the initial soliton energy. We have numerically computed the eigenvalues of the reflected beams [15], and we obtained that they are also spatial solitons but with smaller form factors. The form factor is a parameter which indicates how much the soliton amplitude (width) decreases (increases), while conserving the same area of a first-order soliton.

As is evident from Fig. 3, the form factor of the reflected soliton decreases as we increase the transverse velocity of the incident soliton. This is also shown in Fig. 4, where we plot the form factor of the reflected soliton as a function of the transverse velocity of the incident soliton. Notice that the form factor decreases monotonically with V_{in}. For large enough values of V_{in}, the intensity of the reflected beam is too small, and it will not be able to support a spatial soliton.

The most important quantitative characteristics of the internal reflection of the spatial soliton at our non-linear interface are given in Figs 5 and 6. In Fig. 5 we have plotted the position of the beam centre as a function of the propagation distance for several values of the transverse velocity of the incident spatial soliton. The beam centre is defined in the usual form

$$\hat{x} \equiv \frac{\int a^* X a \, dX}{\int a a^* \, dX} \tag{7}$$

and its dependence with the propagation distance gives us information about the reflection angle, the penetration depth of the incident soliton at the interface, and also about the

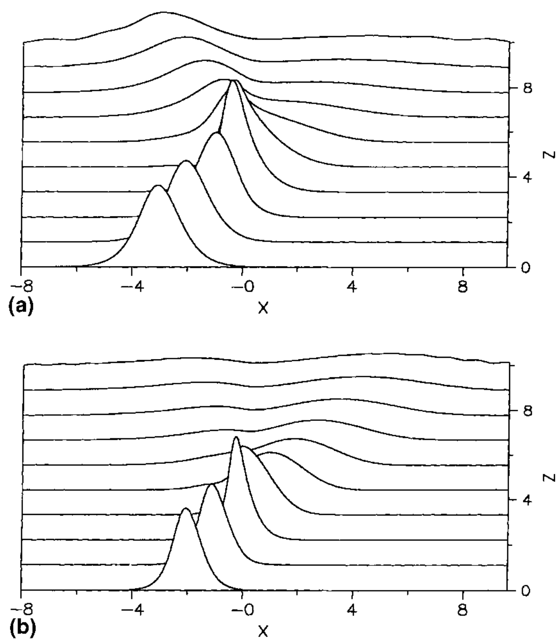

Figure 3 Inelastic internal reflection of a spatial soliton obtained for an initial transverse velocity of (a) $V = 1.5$ and (b) $V = 1.9$. Part of the initial energy is split into the reflected and transmitted beams, but the reflected beams are still spatial solitons.

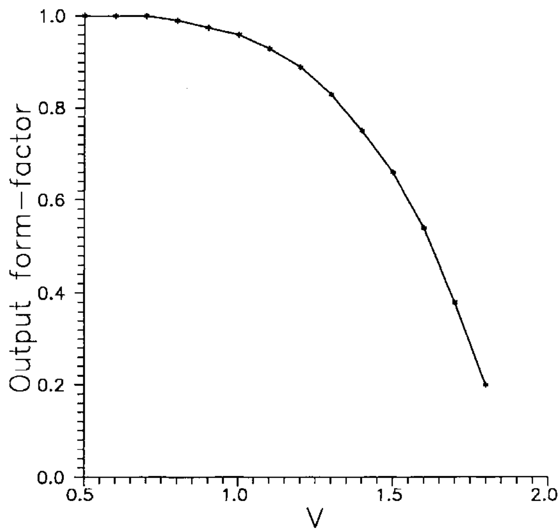

Figure 4 The form factor of the reflected soliton as a function of the transverse velocity of the input soliton.

Goos–Hänchen shift [20, 21]. First, the transverse velocity of the soliton at any propagation distance Z is taken as the Z derivative of the $\hat{x}$ versus Z curve. Figure 5 shows that for $V_{\text{in}} = 1$, the incidence and the reflection angles, taken far away from the incidence point, are essentially the same. However, as the transverse velocity of the initial soliton increases the transverse velocity of the reflected soliton decreases. This important characteristic of the non-linear interface is plotted in Fig. 6, where we see how far away from the classic condition $V_{\text{in}} = V_{\text{out}}$ we are as the incidence angle increases. Notice that the deviation from the classic condition varies in a non-linear way with the initial transverse velocity.

On the other hand, Fig. 5 also shows that the penetration depth of the soliton at the interface increases as V_{in} increases. At the same time, the Goos–Hänchen shift also

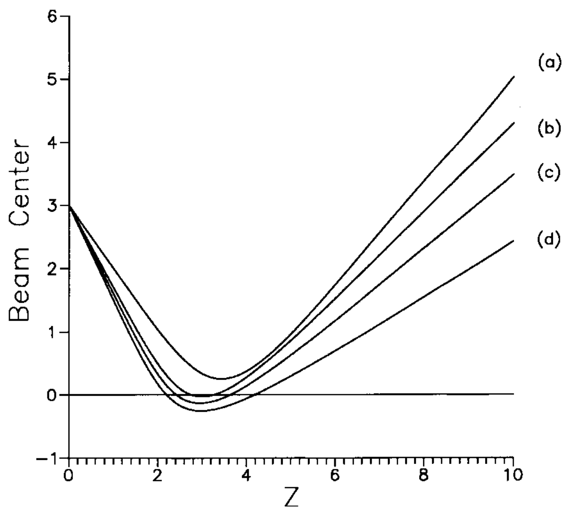

Figure 5 The position of the beam center as a function of the propagation distance for several values of the transverse velocity of the input spatial soliton, (a) $V = 0.5$, (b) $V = 1$, (c) $V = 1.5$, (b) $V = 1.9$, (d) $V = 2.1$.

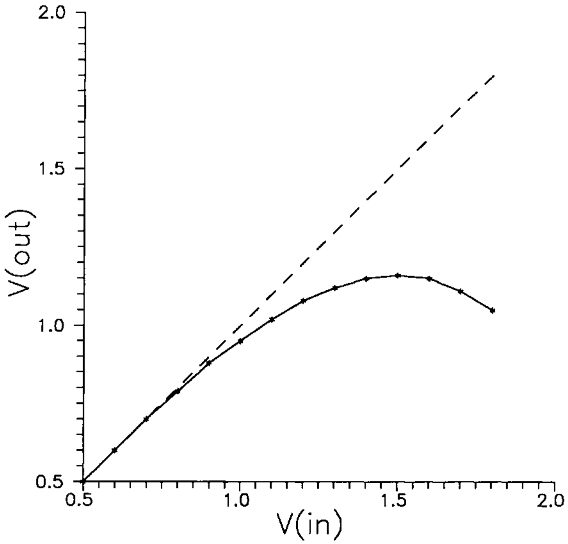

Figure 6 Transverse velocity of the reflected soliton as a function of the transverse velocity of the input soliton.

increases as one increases the incidence angle. We now give an analytical description of the numerical results obtained in this section.

4. Analytical description for the soliton internal reflection at the non-linear interface

Our analytical description of the internal reflection of a spatial soliton at the non-linear interface is based on the particle-like behaviour of the soliton [20, 21]. According to this approach, the most important characteristics of the non-linear interface can be obtained by solving the dynamical equation for the soliton centre. For our modified NLSE, Equation 5, the propagation of the soliton centre, as defined in Equation 7, satisfies the following equation

$$\mathrm{i}P_0 \frac{\mathrm{d}\hat{x}}{\mathrm{d}Z} = \frac{1}{2}\int \left(a\frac{\partial a^*}{\partial X} - a^* \frac{\partial a}{\partial X}\right) \mathrm{d}X \tag{8}$$

where $P_0 = \int aa^*\, \mathrm{d}X$. This equation was obtained by applying the operator $\int a^* X\, \mathrm{d}X$ to Equation 5, and by adding this with the result of applying the operator $\int aX\, \mathrm{d}X$ to the complex conjugate of Equation 5. If we now take the Z derivative of Equation 8 and use again Equation 5, we find that the soliton centre satisfies the equation of motion,

$$P_0 \frac{\mathrm{d}^2\hat{x}}{\mathrm{d}Z^2} = \Delta \int \frac{\partial f(X)}{\partial X} aa^*\, \mathrm{d}X + \frac{1}{2}\int \frac{\partial f(X)}{\partial X} (aa^*)^2\, \mathrm{d}X \tag{9}$$

For this analytical description, we can use $f(X) = U(X)$, and replace $\mathrm{d}f(X)/\mathrm{d}X$ by the Dirac delta function $\delta(X)$. Moreover, we make the strong assumption that the soliton moves like a particle without changing its profile. From the numerical results presented above, this condition is satisfied only for small enough incidence angles when an elastic reflection occurs. However, as we will see later, this assumption allows us to give a quantitative estimation of several important physical parameters. From the mathematical point of view, this assumption means that

$$aa^* = \operatorname{sec h}^2[(X - \hat{x}(Z))] \tag{10}$$

Under this condition, Equation 9 can be rewritten in the following form

$$P_0 \frac{\mathrm{d}^2\hat{x}}{\mathrm{d}X^2} = F(\hat{x}) \tag{11}$$

which establishes the motion of a *particle* $\hat{x}$ of *mass* P_0 under the *force* $F(\hat{x})$ given by

$$F(\hat{x}) = -\Delta \operatorname{sec h}^2(\hat{x}) - \frac{1}{2}\operatorname{sec h}^4(\hat{x}) \tag{12}$$

The trajectory of the beam centre can now be obtained by solving Equation 11 with the initial conditions

$$\left.\frac{\mathrm{d}\hat{x}}{\mathrm{d}Z}\right|_{Z=0} = V_{\mathrm{in}}$$
$$\hat{x}\big|_{Z=0} = -x_0 \tag{13}$$

However, an alternative to solve the Newton's equation of motion, Equation 11, is to use the energy conservation principle. This is possible, because the effective force of Equation 11 can be associated with the effective potential

$$v(\hat{x}) = \Delta[\tanh(\hat{x}) + 1] + \frac{1}{2}\left[\tanh(\hat{x}) - \frac{1}{3}\tanh^3(\hat{x}) + \frac{2}{3}\right] \tag{14}$$

in terms of which the Hamiltonian acting on the beam centre is obtained

$$H = \frac{P_0}{2}\left(\frac{\mathrm{d}\hat{x}}{\mathrm{d}Z}\right)^2 + v(\hat{x}) = \frac{P_0}{2}V_{\mathrm{in}}^2 + v(x_0) \tag{15}$$

Notice here that the first parameter we can estimate is the critical incidence angle at which an elastic internal reflection of the soliton will occur. This is obtained when the kinetic energy of the initial soliton is transformed into potential energy. In other words

$$\frac{P_0}{2}V_{\mathrm{cr}}^2 = v(\infty) \tag{16}$$

or, using Equation 15:

$$V_{\mathrm{cr}} = \left[\frac{4}{P_0}\left(\Delta + \frac{1}{3}\right)\right]^{1/2} \tag{17}$$

This result is in good agreement with our numerical simulations, where we used $P_0 = 2$ and $\Delta = 1$. It is worth pointing out that a first order soliton with higher energy (higher form factor) will have a smaller critical incidence angle.

The penetration length, X_{p}, is obtained from the condition $V = 0$, which is satisfied if

$$X_{\mathrm{p}} = \frac{1}{2}\ln\left(\frac{V_{\mathrm{in}}^2}{V_{\mathrm{cr}}^2 - V_{\mathrm{in}}^2}\right) \tag{18}$$

The trajectory of the soliton centre predicted in Equation 18 allows us to give approximate values for the penetration depth and also for the Goos–Hänchen shift occurring during the elastic internal reflection of the soliton. The Goos–Hänchen shift, ΔZ, is defined as $\Delta Z = (Z_2 - Z_1)$, where Z_1 and Z_2 are the points at which the parabolic-like trajectory of the soliton centre crosses the Z-axis (see Fig. 5). ΔZ can be estimated by assuming a parabolic expansion around the point at which X_p occurs; that is,

$$\hat{x}(z) \approx x_\mathrm{p} + \frac{1}{2}\frac{\mathrm{d}^2\hat{x}}{\mathrm{d}z^2}\bigg|_{x_\mathrm{p}}(z - z_\mathrm{p})^2$$

Noting that

$$\frac{\mathrm{d}^2\hat{x}}{\mathrm{d}z^2}\bigg|_{x_\mathrm{p}} = -\left(\Delta + \frac{1}{2}\right)$$

the Goos–Hänchen shift is estimated to be

$$\Delta z = 2\left[\frac{x_\mathrm{p}}{(\Delta + 1/2)}\right]^{1/2} \tag{19}$$

for $\Delta \gg 1$.

5. Conclusions

We have studied the internal reflection of a bright spatial soliton at a non-linear interface, which divides a Kerr-like and a linear medium. Our study was focused on the possibility of obtaining a reflected spatial soliton. Our results indicate that there are three fundamental cases. In the first case, for $V_\mathrm{in} < V_\mathrm{cr}$, the reflection is an elastic one, and it is possible to give analytical estimations for the penetration depth and for the Goos–Hänchen shift. In the second case, for $V_\mathrm{in} \leq V_\mathrm{cr}$, we have 'inelastic' internal reflection of the soliton. The reflected soliton has a smaller form factor than the incident one, and also $|V_\mathrm{out}| < |V_\mathrm{in}|$. Finally, for large input angles ($V_\mathrm{in} > V_\mathrm{cr}$), a large part of the input energy is transmitted to the second medium in the form of a spreading beam. In this case, the reflected beam does not form a spatial soliton due to its low intensity.

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