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A device approach to propagation in nonlinear photonics crystal

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Abstract

We discuss the propagation in a nonlinear photonic crystal NPC by introducing an effective nonlinear dielectric constant (ENDC). We successfully produce the expected prediction of bistability, switching and changes in the photonic bandgap in a Stack with a Kerr nonlinearity. We demonstrate that the expected nonlinear chirping is not only a characteristic of a nonlinear slab, but of a nonlinear stack as a whole. This ENDC method is quite flexible and susceptible to problems that have a linear solution, as shown with some examples.

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1. Introduction

The electromagnetic (EM) propagation in materials have occupied for long the interest in many fields, in particular in optics and EM communications and in general, in those fields where transmission and propagation are relevant. Nowadays, when we perceive that a new complex structure is important to those objectives, no its matter complexity, we are confident of its feasibility because our mastering of the nanofabrication techniques, that naturally includes nonlinear materials. The search for those new fundamental questions and quite practical applications that goes from switches, logic and all optical storage devices effectively resides in our creativity. That in turn, creates a strong demand for analysis, not sufficed by analytical techniques, that has lead to a growth on numerical capabilities for linear systems of almost any structure.

Propagation in nonlinear photonics crystal (NLPC) [1,2] shares both features, complex PC structures and a nonlinear dynamics, that prevents the linear PC handling of the now strongly coupled counterpropagating waves. However, the same structure allows for the prompt building up of very high intensities and henceforth nonlinear phenomena in small dimension structures. Also, a linear PC is described by its material characteristics, resumed on the photonic band gap structure, that is quite a convenient concept to understand a device. On the other hand propagation in a nonlinear homogeneous media describes precisely a media that is not further independent of the propagating radiation and where the propagating and counterpropagating waves are strongly coupled everywhere in the material. The complexity of a PC structure has been given enough

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support to produce generic codes, often available to the public and even commercial. Nanofabrication has allowed to create almost any structure for a linear PC, and its clear that such a capability will not be an essential restriction on a NPC. Nonlinear propagation in homogeneous media has introduced concepts such as solitons, nonlinear chirping and bistability in media where the nonlinearity is at its core, or where the nonlinearity is at the substratus such as in resonant pulse propagation on the presence of a nonlinearity. Therefore, it is straightforward to question if a NLPC shares also those features and further explore its implications.

As a first step, we shall extend those PC linear concept to understand the field material dependence of the propagation. For this, we explore the most simple PC structure, an stack to model a one dimensional (1D) PC [19], without restricting ourselves to a null reflective signal as was done by Winful et al. [3], in a series of papers that emphasizes the device point of view. Null reflectivity is a very sensible assumption on specific cases [4], but we will focus on its explicit presence. A second, and closely related emphasis are Gap solitons introduced as early as Chen and Mills [5], however, actual solutions were given by Aceves and Wabnitz [6] and Christodoulides and Joseph [7], and are widely reviewed [8], generalized [9] and experimentally demonstrated [10]. The expectation that this systems support soliton solutions is out the core of the approximated methods developed by Goodman [11] and supported by experiments cited there. His nonlinear envelope coupled mode equations govern the coupled backward and forward EM components constructed on the linear plane wave solutions; while the reflected signal is taken into account by a coupling coefficient.

We introduce an iterative method to buildup the nonlinear, Kerr type, solution, more along the lines of an effective dielectric constant.

2. Method

The polarization is described in every slab of the stack by

$$P = \int_{-\infty}^{t} \varepsilon(x, t - t')E(x, t')\,\mathrm{d}t' \quad \text{or} \quad P_\omega = \varepsilon_\omega(x)E_\omega(x) \tag{1}$$

with a frequency Kerr nonlinearity term given by $P_{nl}(x, \omega) = \beta I_\omega$ where $I_\omega = |E_\omega(x)|^2$ is the spectral intensity. The resulting wave equation is given by

$$\frac{\partial^2 E_\omega(x)}{\partial x^2} - \left(\frac{\omega}{c}\right)^2 \varepsilon_\omega(x)E_\omega(x) = 0, \quad \text{where} \quad \eta = \eta_0 + \beta I_\omega \tag{2}$$

The single lossless and nonlinear nth slab is described, Chen [12], by

$$\frac{\mathrm{d}^2 E_\omega}{\mathrm{d}z^2} + (1 + \beta_n \mid E_\omega|^2)E_\omega = 0 \tag{3}$$

where β_n is the nonlinear coefficient of that layer and $z = xk_n$. The proposed solution for the electric field within that layer is:

$$E_\omega(z) = E_0\varepsilon(z)\mathrm{e}^{\mathrm{i}\phi(z)} \tag{4}$$

where [12]:

$$\frac{\mathrm{d}\phi}{\mathrm{d}z} = \frac{w}{\varepsilon^2} \tag{5}$$

This type of solutions has been explored in others areas such as the resonant pulse propagation in two level atoms in the presence of a Kerr substratus. Where w is the well known chirping constant identified by Matulic [13] as the rate of flux along the propagation axis, which correspond to a trivial lineal solution if null. Eq. (5) seems to point out that the intensity dependence of the chirping is a common nonlinear propagation characteristic. The amplitude equation is given by

$$\left[\frac{\mathrm{d}\varepsilon}{\mathrm{d}z}\right]^2 \varepsilon^2 - w^2 + \varepsilon^4\left[1 + \frac{1}{2}\widetilde{\beta_n}\varepsilon^2\right] = a\varepsilon^2 \tag{6}$$

where a is an integration constant. By the change of variable $I_n(z) = \varepsilon^2(z)$, the integration of Eq. (6) can be rewritten as

$$\int_{I_n(d)}^{I_n(x)} \frac{1}{\left(w^2 + aI - I^2 - \frac{1}{2}\widetilde{\beta_n}I^3\right)^{1/2}}\mathrm{d}I = \pm 2k_n(x - d) \tag{7}$$

These elliptic Jacobi functions correspond to the amplitude solution and depend on three parameters w, a and $\widetilde{\beta_n}$. The first two are important constant of motion, in particular w that evidence the most immediate, although not obvious, particularity of a nonlinearity, the chirping; that raises from a single slab and we expect should appear in the stack propagation. Similar integrals have been discussed in great detail in resonant pulse propagation, and clearly include the plane wave solutions when the Kerr nonlinearity is absent. Also, we realize from Eq. (7) that there are two solutions that raises from the plane waves and become distinctive as the nonlinearity increases. We have further explored and compared [11] this integral, by exhibiting in Eq. (4) the plane wave and pointed out on specific solutions [14]. Chen and Mills disregarded the reflection on their solution, but showed intensity dependent chirping as one of the characteristics on the propagation in a nonlinear layer. We shall prove that this is a characteristic of the stack as a whole. The effective dielectric material [15], of proven effectiveness, preserves all the basic elements of the propagation in the stack. Therefore we have developed an effective nonlinear dielectric constant.

3. Nonlinear stack

The linear periodic linear dielectric stack has been thoroughly studied, and the counter propagating waves coupling at the stack interfaces in a is the basis of our physical insight. Its propagation is characterized by a certain range of frequencies where is forbidden, the so called photonic band gap. Those and other phenomena such as trapping of light and the shape of the photonic band gap itself have been explored in the linear domain and become a reliable guide for our experience. The unquestionable validity of the superposition, makes the linear model completely suitable to deal with CW as well as pulsed signals. The methodologies for calculations of the wave dispersion relations have been also presented, under different considerations [16]. The alternating refraction indexes are described by

$$\eta(x) = \begin{cases} \eta_2 & 0 < x < b \\ \eta_1 & b < x < a \end{cases} \tag{8}$$

where $\eta(x + a) = \eta(x)$, x is along the propagation and a is the spatial period (Fig. 1). In the linear case $\beta_1 = \beta_2 = 0$, both η_1 and η_2 are constants. The solution at the ith layer is given by the superposition of the basic linear solutions as [17]:

$$E_i(x) = A\mathrm{e}^{\mathrm{i}k_i x} + B\mathrm{e}^{-\mathrm{i}k_i x} \tag{9}$$

where A is the amplitude of the forward and B is the backward waves, respectively.

The aim is to relate the first and end amplitudes by a transfer Matrix M, that is given in terms of the wave vectors at each layer k_1, k_2 and the refraction indexes η_1, η_2 and the respective thickness of the dielectric layers. The continuity conditions for $E(x)$ and $\partial E(x)/\partial x$ derived from the second order Eq. (3) with $\beta = 0$, at the interfaces $(n-1)a + b < x < na$ and $na < x < na + b$. These equations for the na interface, we have:

$$E_1(na) = E_2(na); \quad \frac{\mathrm{d}E_1(na)}{\mathrm{d}x} = \frac{\mathrm{d}E_2(na)}{\mathrm{d}x} \tag{10a}$$

And in the interface $na + b$, we obtain:

$$E_1(na+b) = E_2(na+b); \quad \frac{\mathrm{d}E_1(na+b)}{\mathrm{d}x} = \frac{\mathrm{d}E_2(na+b)}{\mathrm{d}x} \tag{10b}$$

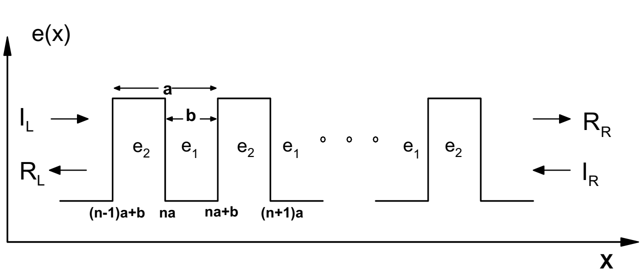

Fig. 1. Refraction index profile as function of the position.

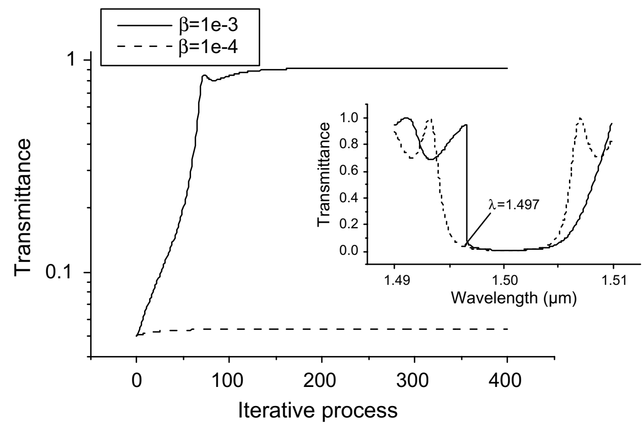

Fig. 2. Convergence of the solution as a function of the wave vector.

From Eqs. (3) and (7), it is clear that we can develop an iterative method providing a sensible support to a common suggestion. Such a method converges to a nonlinear solution, Fig. 2, and it implies to solve the equation.

$$\frac{\mathrm{d}^2 E_\omega}{\mathrm{d}x^2} + \eta^2 E_\omega = 0 \tag{11}$$

with:

$$\eta^l_{N,i} = \eta_{0,i} + \Delta\eta^l_{N,i} \tag{12}$$

$$\Delta\eta^l_{N,i} = (1-\gamma)\Delta\eta^l_{N,i} + \gamma\beta_{N,i} I_{N,i} \tag{13}$$

where $\Delta\eta$ is the intensity dependent change of the refraction index each iteration; γ the correction parameter; l the iteration number η the nonlinear refraction index; i the dielectric medium index in the layer ($i = 1, 2$) N the period number on the stack; $\eta_{0,i}$ the lineal refraction Index of each dielectric medium; β the non lineal material coefficient; and I the electric field mean intensity at each step.

It is clear, that the plane wave method applies at every step of this procedures. The typical convergence, Fig. 2, shows a dependence on β. Also, that the material characteristics (band gap) becomes dependent of the field trough the refraction index and the internal field. That is the coupling of the forward and backward incident waves within the material. We can still force their attachment, Eq. (10), between the counter propagating waves by making a small segmentation of the dielectric (Fig. 3). But that showed little effect. For the method, it is clear that is very convenient to have an analytical solution on the slab, but is enough to have a linear solution, and that might be numerical. We will show some results that are beyond the stack case.

4. Numerical results

We are going to carry the comparison of three different cases. In the first two cases we will consider the lin-

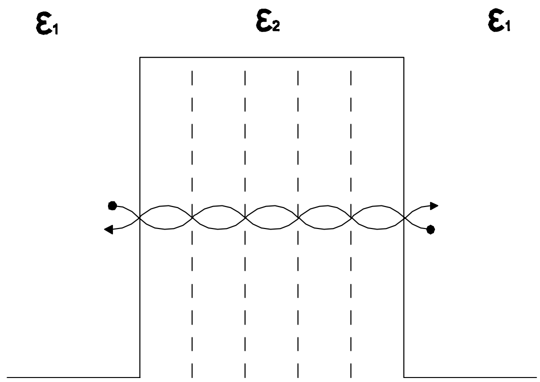

Fig. 3. The nonlinear dielectric is sliced into sublayers of the stack, in order to calculate the coupled field in each interface of the sublayer.

ear refraction index η_0; but in the first case, only the second one is nonlinear $\beta \neq 0$. In the second case, both of them are nonlinear and with the same β. In the last case, we consider two materials that have the same linear refraction index η_0, but only the second one is nonlinear $\beta \neq 0$.

We have chosen the same η_0 for both cases, but in the first case, only the first one is nonlinear Fig. 4a. In the second one, both of them have the same nonlinearity Fig. 4b. In the background we can recognize the linear band gap as a reference, and the increasing nonlinearity produces a small shift of the center of the band gap, but quite a noticeable steeping or red shift of the blue band edge, with the consequent narrowing of the gap. While when both of them have the same nonlinearity, its is evident that the width remains, however such a shift and steeping occurs.

We explore now tuning with the nonlinearity. In a stack with the same linear refraction index for both

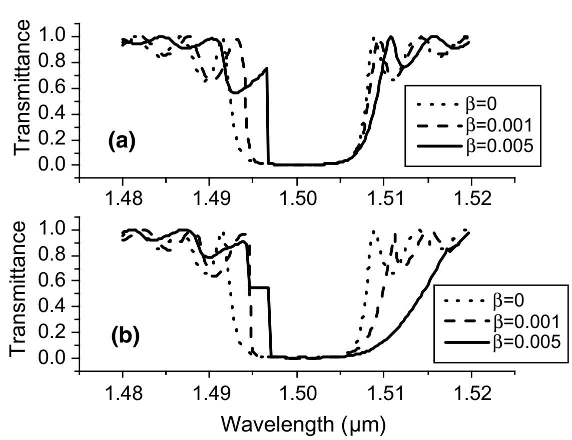

Fig. 4. (a) An alternating linear and nonlinear stack is shown. The nonlinear layers are those of lower refraction, and because the nonlinearity, the refraction indexes difference decreases, and henceforth the width of the transmission band. However is quite noticeable a steppening of the low edge of the gap. (b) The variations of the transmittance band due to the same nonlinearity in both media.

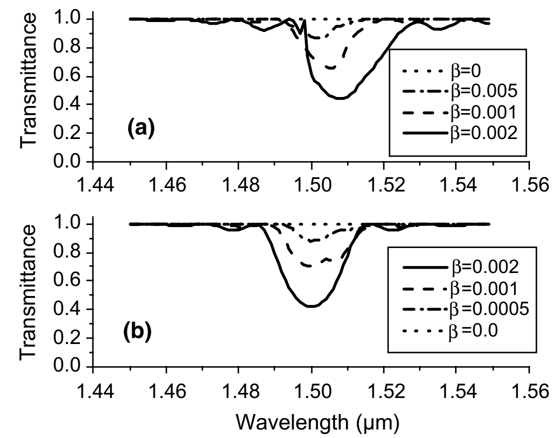

Fig. 5. (a) A partial gap is produced with intensity for various values of β, in a stack with the same linear η_0 for both dielectrics. (b) A nonlinear stack switch is produced with slightly different refraction indexes of very close values, If the lower refraction index is nonlinear, the gap is closed with intensity.

dielectrics, but the second one nonlinear, it is evident, the raise of a switch with over 50% reflection as the intensity grows, and we can notice the expected width growth Fig. 5a. On the other hand, we assume a the slightly different η_0, enough to produce a partial gap (60% reflection), but we can obtain full transmission with the intensity nonlinearity (Fig. 5b).

Other two features that are nonlinear signatures for these systems are the nonlinear switching and the bistability. The nonlinear switching occurs when tuning a pulse in the gap, were transmission is not allowed, by increasing the intensity that occurs. We shall use the case of identical nonlinearity given in Fig. 5a.

In Fig. 6, we display for different intensities the transmission of a pulse at the edge of the photon gap. We can notice a sharp change on the transmitted pulse, due to the change of the intensity (remember that β becomes normalized to the intensity for numerical computations). The transmission is evident in the curve before the last one ($\beta = 1 \times 10^4$), in comparison to the preceding one ($\beta = 1.5 \times 10^4$) that hardly transmits. We can understand this behavior if we notice that the transmission edge on the photonic band gap is displaced toward the center, as the intensity grows.

Herbert [18] experimentally demonstrated that a nonlinear stack shows bistability, as many other nonlinear systems. For the same parameters we have carried out the numerical simulation (Fig. 7). We can notice the excellent coincidence of our results. The paths of growing and decreasing intensities paths are mark down with arrows on the numerical simulation.

A new feature, not commonly predicted, is chirping of a stack (the corresponding to $(\partial\phi/\partial x)$ for a slab, Eq. (6)). Our interest is to find out if indeed is a function of the intensity I. In Fig. 8, we display the phase and its

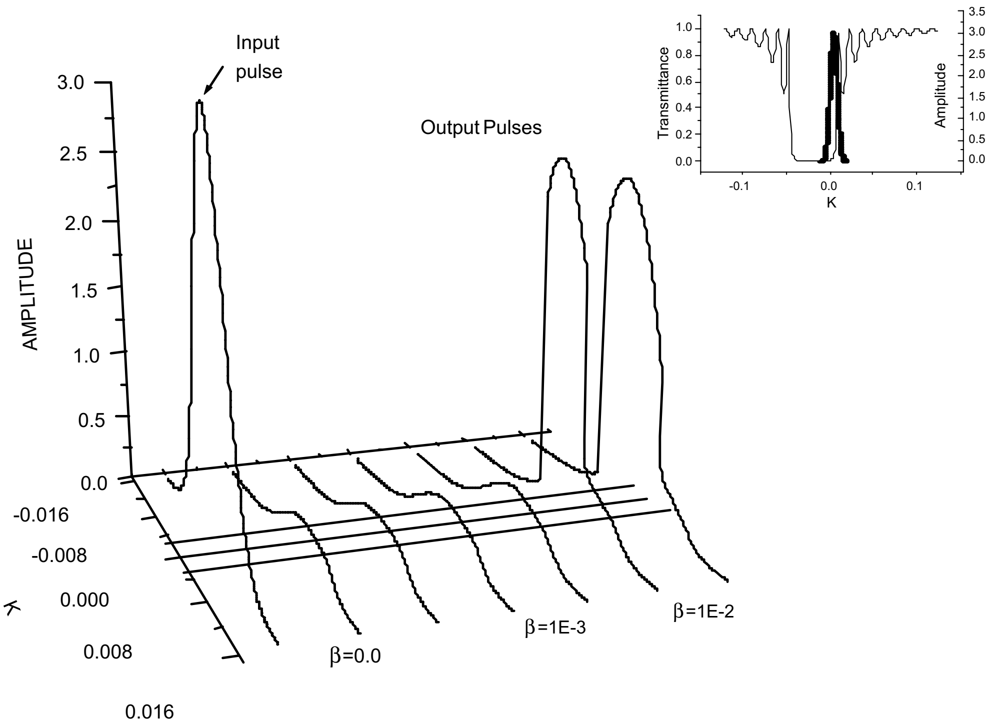

Fig. 6. We show the incident pulse and the transmitted pulse for growing intensities, as a function of the wave number k.

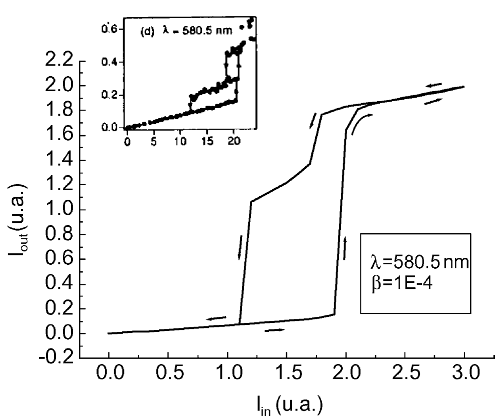

Fig. 7. Our results from the simulation, for the presented case in Fig. 9. The inset shows: the experimental results by Herbert [18] for wave vectors with a modified input power.

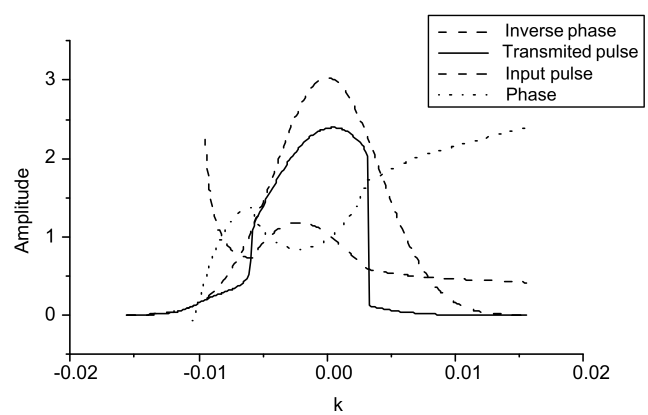

Fig. 8. The phase and its inverse, the transmitted and the incident pulses are shown as a function the of the wave number k.

inverse, as well as the transmitted and incident wave as a function of k. It is rather evident that the inverse of the phase is strongly related to the intensity pulse of the mentioned pulses, and strongly modified by the band gap and its phase changes (notice the sharp turns).

5. Conclusions

Nonlinear Photonic structures are already complex linearly but solvable, but in general, well beyond exact capabilities when is considered a no linearity. A stack is the simplest of and more illustrative photonic band devices. Its nonlinear behavior shows a rich nonlinear dynamics, such as chirping, bistability, and switching that can be easy modeled with the using of an nonlinear effective dielectric constant. The method is quite flexible, and can be use in many other systems and more complex NLPC, as long as a linear solution exists both analytical or numerical. In Fig. 9 we show the radial band gap of an omniguide fiber, detail discussion is published elsewhere.

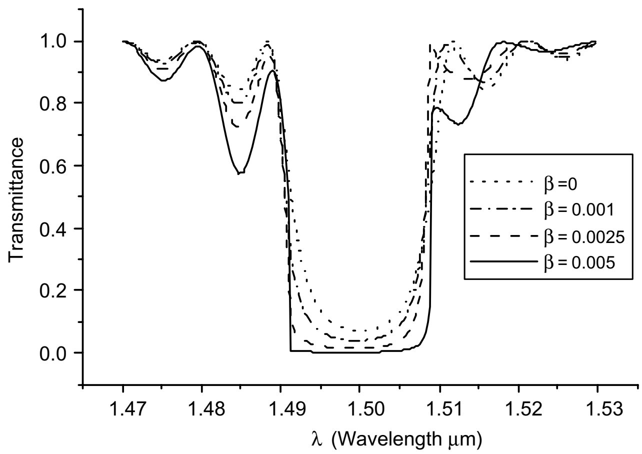

Fig. 9. Transversal propagation in a cylindrical multilayer system, where the nonlinear behavior of the structure is shown.

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