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INVITED PAPER

# Optical spatial solitons

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It is almost common knowledge now that particles also possess properties of waves. But, can localized wave-packets behave and interact like particles? This article is dedicated to self-trapped optical beams: how they form and how they interact with each other, showing that their particle-like behaviour is a general feature that manifests itself in many other nonlinear systems in nature.*

## 1. Introduction

Although mankind has always been fascinated by visual manifestations of nonlinear wave phenomena such as tsunamis, tidal waves etc., the first scientifically documented report of self-trapped wave-packets was in 1834, when a Scottish scientist, John S. Russell, observed a ‘rounded smooth and well defined heap of water’ propagating in a narrow and shallow canal ‘without change of form or diminution of speed’ [1]. The water was calm on both sides of this, rather unusual form of ‘wave’, and Russell noted that the wave had the form of a ‘solitary elevation’. The phenomenon intrigued Russell and he noted it in his note-book as a ‘great primary wave of translation’. It took more than fifty years for two Dutchmen, Korteweg and de Vries, to realize that for this phenomenon to occur the ‘solitary wave’ must have an unusually large amplitude. This means that the medium in which the wave propagates (water, in this case) must behave in a fundamentally different manner to waves of different amplitudes, that is, its behaviour is *nonlinear*. During the following seventy years similar phenomena have been observed in many other systems in which waves propagate, such as charge density waves in plasma and phonons in solids, but

* The purpose of this article is to introduce the area of optical spatial solitons to the non-expert. It is written in the spirit of physics at the finger-tips, and is intended for researchers that wish to understand the main ideas in this new and exciting field.† A short summary of this paper will appear as a review article in *Physics Today*, co-authored with George Stegeman, of CREOL, University of Central Florida. I have kept the same abstract as in the *Physics Today* paper, because it conveys the main message: optical spatial solitons have become a playground for exploration of deep and rich phenomena that can be found in almost any non-linear system in nature: solitons.

† A reader interested in a detailed review of the theory of Kerr and non-Kerr spatial solitons is encouraged to read the review articles by Yuri Kivshar and Nail Akhmediev in this issue; I have read them both and they are exceptionally well written.

 

they were considered little more than a curiosity. In 1965, however, Zabusky and Kruskal [2] realized that localized wave-packets (self-trapped pulses), under certain assumptions about the form of the nonlinearity, maintain their identities even when they undergo collisions with each other, and that each one of them conserves its power and initial velocity. They concluded that these pulses behave and interact with each other like particles do, and named them 'solitons'. Soon thereafter an immense amount of theoretical and experimental work followed and the general features of solitons were observed in many different branches of science. This article is dedicated to one particular type of soliton which had been almost dormant for several decades but has experienced a mini-revolution in the last few years: namely the *optical spatial soliton*.†

The fact that soliton wave packets do not spread imposes unusual constraints on the wave motion. Pulses (wave-packets) in nature have a natural tendency to broaden during propagation in a dispersive linear medium. This feature is observed in many different systems in which waves propagate, such as density waves in fluids, charge waves in condensed matter, and electromagnetic waves in media with small (finite) absorption. In optics, a localized pulse in space or in time can either broaden in its temporal shape, its spatial extent, or both. For temporal pulses this broadening (temporal lengthening) is due to chromatic dispersion: the various frequency components that constitute the temporal pulse possess different velocities (due to the presence of some, usually distant, resonance). The narrowest pulse forms when the relative phase among all components is zero. However, as the pulse starts to propagate, the frequency components travel at different phase velocities. Hence their relative phase is no longer zero and the pulse broadens. For 'pulses' in space (so-called 'beams'), the broadening is caused by diffraction. Consider a quasi-monochromatic light beam propagating within a medium of refractive index $n$ in some (arbitrary) general direction that is called the 'optical axis', for example along $z$. The beam can be represented as a linear superposition of plane-waves (sometimes called 'spatial frequencies'), all having the same wavevector ($k = n\omega/c$, i.e. the ratio between the frequency and the speed of light $c/n$), with each wave propagating at a slightly different angle with respect to the optical axis. Since each plane-wave component is characterized by a different projection of its wavevector on the optical axis, each component propagates at a different phase velocity with respect to that axis. In this way, the component that propagates 'on' the optical axis propagates faster than a component that propagates at some angle $\alpha$, whose propagation constant is proportional to $\cos(\alpha)$. Just as for temporal pulses, the narrowest width of the spatial beam is obtained at a particular plane in space at which all components are in-phase. However, as the beam propagates a distance $z$ away from that plane, each plane-wave component '$i$' acquires a different phase (equal to $[2\pi nz\cos(\alpha_i)]/\lambda]$, $\lambda$ being the wavelength in vacuum). This causes the spatial frequency components to differ in phase and the beam broadens (diffracts). In general, the narrower the initial beam, the broader is its plane-wave spectrum (spatial spectrum) and the faster it diverges (diffracts) with propagation along the $z$-axis. A commonly used method to eliminate spatial spreading (diffraction) is to use waveguiding. In a waveguide, the propagation behaviour of the beam in a high index medium is modified by the total internal reflection from boundaries with media of lower refractive index, and under

† The term 'soliton' is used in conjunction with self-trapped wave packets, i.e. I use the broader definition of solitons that includes these in non-integrable systems, as spelled out first by Russell in 1934 and recently defined by Zakharov and Malomed [3].

conditions of constructive interference between the reflections the beam becomes trapped between these boundaries and thus forms a 'guided mode'. A *planar* dielectric waveguide is an example of such a waveguiding system, typically called $(1+1)$ D (or 1D), because propagation occurs along one coordinate (say, $z$) and guidance along a single 'transverse' coordinate ($y$). The guided optical beam is here assumed to be uniform in the other transverse direction $x$. This is equivalent to Russell's spatial solitary wave case which is also $(1+1)$ D with the water displacement occurring along one spatial coordinate ('height'). An optical fibre is an example of a $(2+1)$ D waveguiding system, in which spatial guidance occurs in both transverse dimensions.

Some materials possess considerable optical nonlinearities, that is, their properties (refractive index or absorption) are modified by the presence of light. Obviously, if a nonlinearity is introduced, the propagation of optical pulses (in space or time) can be altered from that of a 'linear' material. In particular, if the medium modifies its refractive index in the presence of light, under certain conditions it is possible to eliminate the spatial or temporal pulse broadening altogether. This occurs when the linear dispersion or diffraction effects are counteracted by phenomena which act like 'light-induced lensing'. This effect eliminates the phase differences that are accumulated during propagation between the components composing the beam thus allowing a non-diffracting or non-dispersing beam to propagate. Light-induced lensing allows the propagation of short temporal pulses that do not change shape as they propagate despite being propagated in a dispersive material (for example, an optical fibre). These non-dispersing pulses are called 'optical temporal solitons'. They were first predicted by Hasegawa and Tappert [4] in 1973 and first observed experimentally by Mollenauer *et al.* [5] in 1980. Temporal optical solitons have generated great interest during the last two decades and are now being considered as a possible candidate for long distance optical communication systems [6, 7].

In principle, one can also use an optical nonlinearity to confine a spatial pulse (a narrow optical beam) without using an external waveguiding system. Intuitively, this can occur when the optical beam modifies the refractive index in such a way that it generates an effective positive lens, i.e. the refractive index in the centre of the beam becomes larger than that at the beam's margins. The medium now resembles a graded-index waveguide in the vicinity of the optical beam. When the optical beam that has induced the waveguide is also a guided mode of the waveguide that it induces, the beam's propagation becomes stationary, that is, the entire beam propagates as a whole with a single propagation constant. *All the plane-wave components that constitute the beam propagate at the same velocity*. As a result, the beam becomes 'self-trapped' and its divergence is eliminated altogether, keeping the beam at a very narrow diameter which can be as small as 10 vacuum wavelengths. For example, a top view photograph of a 10 μm wide spatial soliton propagating in a photorefractive crystal is shown in Fig. 1 (top). For comparison, the same beam diffracting naturally when the nonlinearity is 'turned off' is shown at the bottom of Fig. 1.

## 2. Kerr-type spatial solitons

Being a highly nonlinear problem, the mathematical foundations of solitons pose a rather difficult theoretical challenge. In some simple cases, however, the problem is soluble exactly. One such example occurs when the nonlinearity involved is of the simplest type that can be found in almost any system in nature: *weak symmetric anharmonicity*. As discussed in Appendix 1, the motion of the electron leads to a refractive index, $n$, of the form $n = n_0 + n_2|E|^2$, where $n_0$ is the background refractive index, $\vec{E}(\vec{r}, \vec{t})$ is the electric field

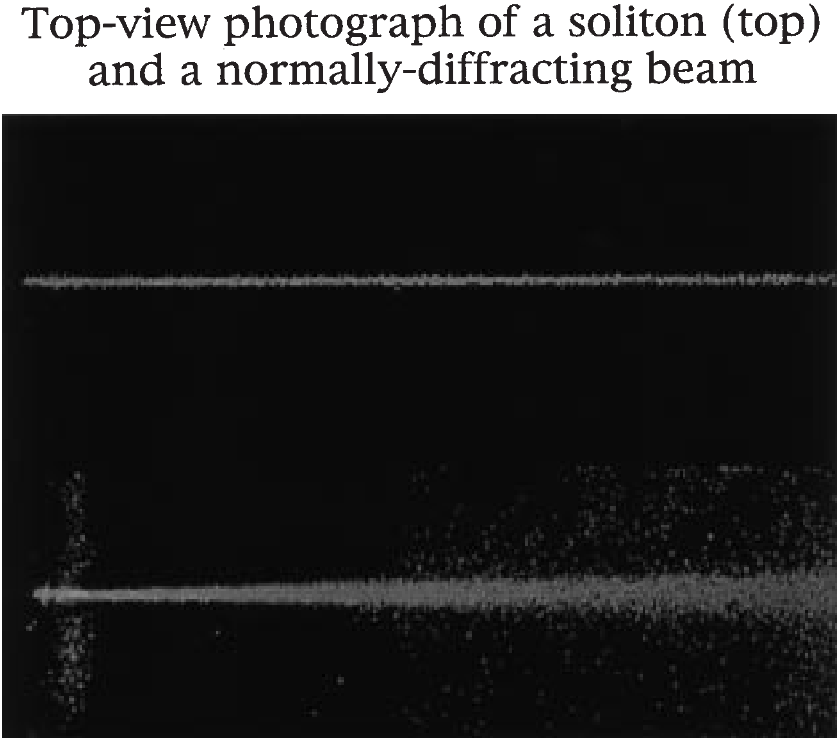


*Figure 1* A top view photograph of a 10 μm wide spatial soliton propagating in a 5 mm long photorefractive crystal (top), and, for comparison, the same beam diffracting naturally when the non-linearity is 'turned off' (bottom). Photograph taken from [35].

amplitude of the EM wave, and $n_2$ is some coefficient whose sign depends on the actual anharmonicity. This produces the self-lensing effect needed for spatial solitons.

Following the first observation of the self-focusing of laser light [8], Chiao *et al.* [9] have shown theoretically in the same year that the governing equation is nothing but the nonlinear Schrodinger (NLS) equation with a cubic potential as just discussed, and that an optical beam propagating in such a one-dimensional nonlinear Kerr medium can indeed self-trap. Several years later, Zakharov and Shabat [10] solved the full $(1+1)$ D problem analytically using the inverse-scattering method. It turns out that solutions to the NLS equation possess some properties that are rather unique in nonlinear dynamic systems. One of them is integrability, which has a consequence that not only do the solitons conserve power and velocity upon collision, but also the number of solitons is conserved and all the interactions amongst solitons are fully elastic. Another important property of bright Kerr-type solitons is that an additional transverse dimension into which the beam can diffract $(2+1)$ D leads to instability. It turns out that when a circular beam is launched into a Kerr-like nonlinear medium, a self-trapped beam is an unstable solution of the $(2+1)$ D NLS equation. Instead of forming a circular soliton, the beam undergoes catastrophic self-focusing and eventually breaks up [11]. *Indeed, many early experiments in nonlinear optics showed this catastrophic self-focusing, frequently leading to damage* [8]. This question of stability can be understood in terms of the characteristic lengths affecting the propagation. For a narrow light beam propagating in either a (1+1) D system (a planar waveguide of thickness $h$) or (2+1) D system (bulk medium), the characteristic diffraction length is given by $L_D = \pi a^2 n_0/\lambda$ where '$2a$' is the beam width along the diffracting co-ordinate(s). Note that the beam in the planar waveguide is initially narrow in the lateral $x$-direction, in which it diffracts as if it were in a homogeneous medium, whereas it is guided in the $y$-direction, so the self-trapping problem is reduced to $1+1$ $(x, z)$ dimensions only. Counteracting this diffraction is the self-focusing nonlinearity whose characteristic length $L_{NL} = a/[\Delta n(|E|^2)/n_0]^{1/2}$ depends on the dimensionality ($L_{NL}$ is calculated from the critical angle for total internal reflection in the induced waveguide; it is also possible to calculate it from Fermat's principal, which gives a similar result). For the Kerr case, $\Delta n(|E|^2) = n_2 P/(ah)$ and $= n_2 P/(\pi a^2)$ for the planar waveguide and bulk media cases respectively, where $P$ is the peak power ($P/(ah)$ and $P/(\pi a^2)$ are simply the beam intensities in both cases. When these two effects balance, the $(1+1)$ D case gives $aP =$ constant, which implies that fluctuations in the power can be compensated by changes in the beam width and vice versa. This property is responsible for the 'robustness' of solitons, i.e. they are highly stable and any optical excitation with parameters close to the soliton parameters usually evolves with propagation into a stable soliton. However, for the 2D case, $P_{cr} = \lambda^2/(\pi n_0 n_2)$ is a constant which means that for $P > P_{cr}$ catastrophic self-focusing occurs, and for $P < P_{cr}$ the beam diffracts. Note that $P_{cr}$ is *independent of the beam size*, which means that variations in the peak power result in instability and not self-trapping. One might think, perhaps, that beams that are very narrow in one dimension, uniform in the other and propagate along the third direction would form stable solitons in the form of one-dimensional (stripe) beams self-trapped in a 3D bulk medium. But such beams are also unstable, as shown by Zakharov and Rubenchik [12] the 'stripe' beam disintegrates into multiple filaments and becomes 'transversely unstable'. The end result is that Kerr-type solitons are stable only in $(1+1)$ D, i.e. one longitudinal dimension along which the beam propagates and one transverse dimension in which the beam diffracts or self-traps. This means that Kerr-type spatial optical solitons can be observed in single-mode waveguides but not in the bulk. These instability properties are universal and are exhibited by *all* Kerr solitons in nature. With this knowledge in hand, Barthelemey *et al.* [13] were able to observe spatial optical Kerr solitons in liquid $CS_2$, by employing an interference grating to 'arrest' the transverse instability. Several years later, Aitchison *et al.* [14] at Belcore observed true $(1+1)$ D Kerr solitons in a single-mode glass waveguide. Soon thereafter the first observations of interactions between spatial solitons (soliton collisions) were demonstrated by both groups, and the elastic collision properties of Kerr-type solitons have been confirmed [15–17]. At that point, it had seemed as if Kerr-like solitons were well understood and other kinds of self-trapped beams (especially $(2+1)$ D solitons) less likely to exist.

## 3. Insights into self-trapping: saturable media

In contradiction to this more-or-less established consensus that two-dimensional solitons are unstable in nonlinear media (based on the results for Kerr media) there stood alone one contradictory experiment. In 1974 Bjorkholm and Ashkin [18] of Bell Labs were able to demonstrate self-trapping of a *circular* laser beam in atomic (sodium) vapour, in the close vicinity of a resonant transition. They were able to find the specific conditions (frequency, beam diameter and intensity) at which the beam self-traps, at least for some finite propagation distance. They conjectured that the effects are due to the *saturable* nature of the optical nonlinearity. It is fair to say that as early as 1969, Daws and Marburger [19] performed 'computer studies in self-focusing' and found numerically that saturable nonlinearities are able to 'arrest' the catastrophic collapse and lead to stable self-trapping of two-dimensional beams. Other authors have reached similar conclusions in several other forms of saturable nonlinearities (related to solitons in plasmas) [20–22]. However, saturable nonlinearities give rise to non-integrable propagation equations, implying that inverse-scattering approaches cannot be used, which, in turn, has made the theoretical predictions far from straightforward. Nevertheless, issues such as stability can be understood easily in terms of the characteristic lengths discussed above since index saturation requires higher order (than $n_2$) nonlinearities to arrest the linear increase in index, i.e. $\Delta n(|E|^2) = \Delta n_{\mathrm{sat}}|E|^2/[|E|^2 + I_{\mathrm{sat}}] = n_2|E|^2 - n_4|E|^4 + \cdots$, where each of these higher order nonlinearities can be (loosely) linked to one of the higher order force constants in the spring expansion discussed in Appendix 1. Retaining just the first higher order term $n_4$ is sufficient to demonstrate stability in the $(2+1)$ D case. Again equating the (modified) nonlinear length to the diffraction length gives $P[1 - n_4P/(n_2\pi a^2)] = P_{\mathrm{cr}}$. As long as $1 \gg n_4P/(n_2\pi a^2)$, i.e. the index change remains positive (which is clearly the case in saturating media when all the higher order terms are included), the beams are stable because $\mathrm{d}P/\mathrm{d}a < 0$.

In succeeding years, however, experimental groups have focused more on temporal solitons, and the experimental field of spatial solitons was deserted until the early nineties (with the exception of some beautiful experiments in liquid $CS_2$ by Barthelemy *et al.* [13] in 1985, and in glasses by Aitchison *et al.* [14] in 1990).

The l990s have been witness to a resurgence of interest in the theoretical aspects of spatial solitons. In 1991, Snyder's group developed an intuitive 'self-consistency' approach which applies to a large family of solitons in nature. They drew on an idea first advanced by Askar'yan [23] in 1962: a soliton forms when an optical beam induces a waveguide (via the nonlinearity) and at the same time is a guided mode of the waveguide it induces. Snyder *et al.* [24] developed this idea into a methodology which offers much insight into the dynamics of spatial solitons, their stability and interactions [25]. This principle can be rephrased in more general terms that apply to a large family of solitons in nature. A soliton (self-trapped wave-packet) forms when the 'pulse' modifies the potential in such a way that the pulse itself is a bound solution of that potential. Following this approach, one can easily understand why self-trapped circular beams can indeed be stable in a saturable nonlinear medium. Saturation of the nonlinearity implies that there exists a maximum value for the optically induced change in the refractive index, for example of the form given above $\Delta n(|E|^2) = \Delta n_{\mathrm{sat}}|E|^2/[|E|^2 + I_{\mathrm{sat}}]$, so that as $|E|^2 \gg I_{\mathrm{sat}}$, $\Delta n(|E|^2)$ asymptotically approaches $\Delta n_{\mathrm{sat}}$. Just as for Kerr media, a saturable medium acts as a focusing lens at high intensities and the beam tends to self-focus so that the power becomes distributed over a smaller area, thereby increasing the intensity. However, because the index change cannot exceed $\Delta n_{\mathrm{sat}}$, the induced lens eventually becomes wider instead of stronger and has less focusing power at its centre. This means that the 'run-away' interaction that gives rise to catastrophic collapse in Kerr media (a consequence of not being able to stop the self-focusing process) can be fully arrested if the nonlinearity is saturable. Another elegant implication is that, since the induced waveguide becomes broader and broader with increasing intensities (rather than deeper and deeper, as in Kerr media), its numerical aperture increases and it is capable of guiding more than one guided mode (the induced potential well becomes broader and more bound solutions can be found). With this intuitive theoretical tool in hand, some experimental groups have launched experiments with saturable nonlinearities, for example Luther-Davies' group at the Australian National University.

### 3.1. Discoveries of new kinds of spatial solitons

In the early nineties, several intriguing events had started to attract interest in spatial solitons. It just so happened that roughly at the same time new types of solitons, each in a nonlinearity of a saturable nature, were discovered and demonstrated experimentally: *photorefractive solitons* and *quadratic solitons*. Both of these new types of solitons exist in $(2+1)$ D and give rise to a whole new 'family' of soliton interactions in three dimensions that were not possible before and a variety of other rich phenomena. Yet another interesting different direction toward which spatial solitons have evolved in the early nineties is *dark* and *vortex solitons* (which are $(2+1)$ D dark solitons). In the sections below each of these types of solitons is discussed separately. This is followed by a section on soliton interactions, which shows that interaction properties are general features common to all soliton phenomena and depend only on whether or not the nonlinearity is saturable, but does not depend much on its explicit form.

## 4. Photorefractive solitons

Photorefractive materials have been known for almost 30 years now [26, 27]. Typically, these are dielectric (or semi-insulating) single crystals that are non-centrosymmetric, that give rise to $\chi^{(2)}$ nonlinearities (whose properties are discussed in Appendix 1). Through the electro-optic effect, a low frequency electric field ($E^{(0)}$) modifies the index of refraction in the medium as $\Delta n = -n^3 r_{\mathrm{eff}} E^{(0)}/2$, where $r_{\mathrm{eff}}$ is the relevant term (determined by the directions of $E^{(0)}$ and the polarization of the optical beam) from the electro-optic tensor. Photorefractive materials always have some 'foreign' atoms (dopants) hosted in the crystalline matrix, with energy levels deep inside the forbidden gap. These dopants are in the form of donors and acceptors, i.e. they can contribute (or trap) free charges. Consider now an optical beam incident upon such a crystal, with optical photons that are not energetic enough to cause valence-to-conduction band excitation, but can excite charges (say, electrons) from the deep dopant levels. Once excited into the conduction band, the electrons are free to move. If the intensity of the optical beam is not uniform in space, these photo-excited electrons experience transport: they diffuse from high concentrations to lower ones and they can drift if an external bias field is applied. At the same time, the donor dopants which are now positively charged are localized immobile ions. Eventually, after some characteristic time (dielectric relaxation time), the electrons are re-trapped (either by acceptors or by ionized donors) at locations that are *different* from their original donor ions. The resulting charge separation establishes an electric field *within* the medium, which *varies in space* (e.g. the field in an illuminated spot is different from that in a dark spot). This internal space charge field, $E_{\mathrm{sc}}^{(0)}$, gives rise to a change in the refractive index via the electro-optic effect. However, since $E_{\mathrm{sc}}^{(0)}$ varies in space, the refractive index $n + \Delta n$ varies in space as well. In other words, non-uniform illumination incident upon a photorefractive medium results in a non-uniform change in the refractive index. In this way photorefractive materials are used to record volume holograms for applications such as optical data storage, realization of phase conjugate mirrors, etc. In the usual context, photorefractive nonlinear optics deals with a periodic interference grating whose envelope is (slowly) modulated by some pictorial information. A soliton is a different 'animal': it entails self-action of a single beam and has nothing to do with holography.

The existence of photorefractive solitons was first predicted at CALTECH in 1992 [28] and demonstrated a year later at the University of Arkansas [29]. Over the six five years, several different types of photorefractive solitons have been discovered, each resulting from a different nonlinear mechanism which is inherently saturable, and each exhibiting a different dependence of $\Delta n$ on the optical intensity $I = |E|^2$. Here, one particular type of photorefractive soliton is described here in detail: the *photorefractive screening soliton*.

Intuitively, one may view the formation of bright photorefractive screening solitons [30] as illustrated in Fig. 2. Consider a narrow light beam propagating in the centre of a photorefractive crystal across which a voltage has been applied. In the illuminated region the density of free electrons increases, which means that the conductivity increases and the resistivity decreases. Since the resistivity is now not uniform across the crystal, the voltage drops primarily in the dark regions (a simple voltage-divider) and this leads to a large space charge field, $E_{sc}^{(0)}$, there and to a much lower field in the illuminated region. (In practice, the spatial distribution of the field in this intuitive voltage-divider picture is supported by different concentrations of ionized donors on either side of them beam; this leads to an electric field 'under' the bean with polarity opposite to that of the applied field, thus resulting in partial screening of the applied field in the region of the optical beam.) The refractive index changes by $\Delta n \propto E_{sc}^{(0)}$ (via the electro-optic effect). If $\Delta n > 0$, this process results in an anti-guide (a large increase in the index in the dark regions) which would strongly defocus the beam. However, when $\Delta n$ is *negative*, the large *negative* index change in the dark regions creates a 'graded index waveguide' that guides the beam that has generated it. (The sign of $\Delta n$ depends on the direction of $E_{sc}^{(0)}$ with respect to the principal axes of the non-centrosymmetric photorefractive crystal, and can be reversed by simply reversing the voltage polarity.) When the optical beam is also the first guided mode

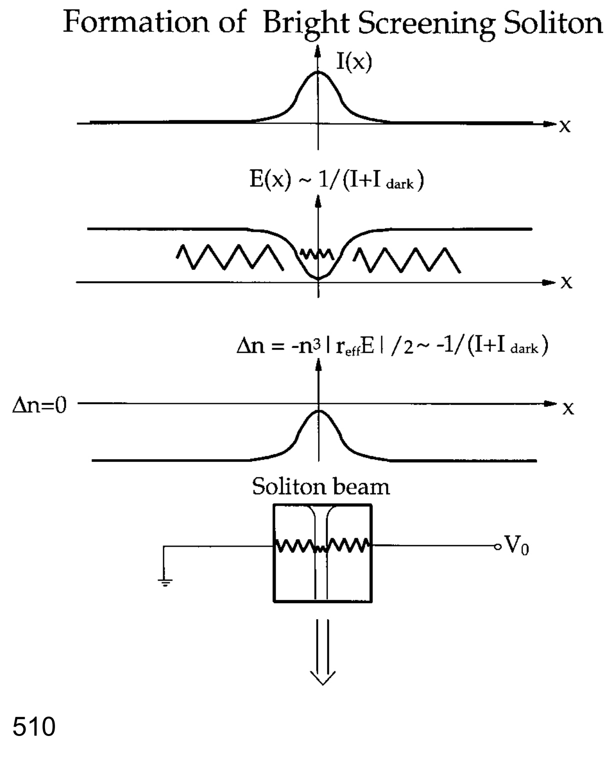


*Figure 2* Formation of a bright photorefractive screening soliton.

of this induced waveguide, the beams' propagation becomes stationary and diffraction is eliminated (self-consistency principle). The actual dependence of $\Delta n$ on the optical intensity $I = |E|^2$, for $(1+1)$ D screening solitons, is $\Delta n = (V/L)(n^3 r_{\text{eff}}/2)[1/(|E|^2 + I_{\text{dark}})]$. Here $r_{\text{eff}}$ depends on the direction of the applied field and the polarization of the beam, $V$ is the voltage applied between electrodes separated by distance $L(L \gg$ soliton width), and $I_{\text{dark}}$ is the so-called 'dark irradiance', which is a material parameter that is proportional to the conductivity of the crystal in the dark.

The subsequent evolution of the photorefractive soliton family has been meteoric. *Photorefractive screening solitons* were first predicted by Segev *et al.* [31] Yariv in 1994, and independently by Christodoulides and Carvalho [32] in 1995. The observation of screening solitons was preceded by a report of steady-state self-focusing effects in biased photorefractive media made by Stepanov's group [33] (from INAOE, Mexico) in 1994. Shortly thereafter, the first observation of screening solitons was made by Shih *et al.* [34, 35] at Princeton. Amongst all types of photorefractive solitons discovered, this type has, by far, attracted most of the experimental attention simply because it is the easiest to understand intuitively in $(1+1)$ dimensions. The difference between photorefractive solitons and Kerr solitons was apparent right from the first observations [29, 34, 35]. It became clear that photorefractive solitons (in general) are stable in $(2+1)$ dimensions as well as in $(1+1)$ dimensions. For example, $(2+1)$ D screening solitons occur when a circular beam causes 'bending' of the electric space charge field at the vicinity of the beam, and under some specific conditions (of beam intensity relative to $I_{\text{dark}}$, beam diameter, and applied field), the field component that gives rise to $\Delta n$ attains an approximately circular symmetry. Another important fact is that all of the observations of *bright and dark* $(1+1)$ D photorefractive solitons were carried out in a 3D bulk medium and do not exhibit 'transverse instability' (at least for any physically-realistic propagation distance of several centimetres) as long as the width and peak intensity of the beam and the applied field are the proper set of parameters that can support a soliton (see, for example, [36, 37]). The transverse instability issue initially has steered much controversy, as it has been initially claimed that $(1+1)$ D photorefractive solitons are unstable in 3D media even for very short propagation distances [38]. A recent theory paper by Infeld *et al.* [39], (see, in particular, the conclusion section), however, has proven that in both bright and dark $(1+1)$ screening solitons the transverse instability is 'arrested' if the parameters of the input beam are close enough to those that support a soliton: just like the experiments preceding that paper have shown.†

As noted above, several other types of photorefractive solitons have been found thus far. '*Quasi-steady-state*' solitons, which exist during a finite window in time (never surviving to steady state) were observed first [29]. They occur when an externally-applied field is slowly being screened by the space charge field. Another kind is the *photovoltaic soliton*. It does not require an external bias field but instead relies on the bulk photovoltaic effect to create the space charge field, which in turn, modifies the refractive index and gives rise to a soliton. The nonlinearity that supports $(1+1)$ D photovoltaic solitons is of the form

† The discrepancy between the theoretical results of [39] and [38] has been resolved in a rather unusual manner. Reference [39] has proved that the results of [38] do not apply for isolated solitons, but for a periodic train of solitons: they are a numerical error resulting from the choice of periodic boundary conditions. The contradiction between the experimental results of [38] (showing transverse instability) and the many experiments demonstrating that these solitons are transversely stable (for example [36, 37]), result from misinterpretation at the experimental data of [38]. See discussion on this issue in [37] (in particular, [44] in that paper).

$\Delta n \propto [|E|^2/(|E|^2 + I_{\mathrm{dark}})]$. Photovoltaic solitons were predicted in 1994 [40, 41], and first observed a year later [42, 43]. A fourth type of photorefractive soliton exists in biased photorefractive semiconductors, such as InP, in which both electrons and holes participate in the formation of the space charge field. Interestingly enough, the self-focusing effects that support these solitons undergo a large enhancement when the rate of optical excitation of holes is close to (but smaller than) the thermal excitation rate of electrons. When the optical excitation of holes exceeds the thermal excitation rate of electrons, self-focusing turns into self-defocusing, i.e. the sign of the optical nonlinearity can be reversed by all optical means. These solitons were discovered experimentally in 1996 by Salamo's group at the University of Arkansas [44, 45]. Finally, recent theoretical work has predicted [46] the existence of solitons in centrosymmetric photorefractive media, which fundamentally do not possess quadratic nonlinearities. The change in the refractive index that gives rise to these solitons is driven by the dc Kerr effect, which is similar to Pockels' effect but $\Delta n$ is now proportional to $(E_{\mathrm{sc}})^2$ and thus to $1/(|E|^2 + I_{\mathrm{dark}})^2$. Recent experiments performed at the Burdoni Institute and the University of L'Aquila (Italy) have demonstrated these solitons [47].

It is worth noting two additional properties that are common to all photorefractive solitons. The first is the ability to generate solitons with optical power levels of 1 μW and lower [34, 35]. This occurs because $\Delta n$, depends on the ratio $|E|^2/I_{\mathrm{dark}}$ rather than on the absolute value of the optical intensity $|E|^2$, and $I_{\mathrm{dark}}$ is typically very low in photorefractive materials (the dark current is very low). The drawback (in terms of applications) is that the response time (dielectric relaxation time) scales as the inverse of the optical intensity, and can be long (seconds) for these power levels in 10 μm wide solitons. The other property is that the materials' response is wavelength dependent. Thus, one can generate solitons with microwatts power and use the waveguides induced by these solitons to guide, steer and control *powerful* (watts) beams at wavelengths for which the material is less photosensitive [48, 49].

## 5. Quadratic solitons

Quadratic solitons are a different breed of soliton from those previously discussed. First, the optical fields do not modify any of the properties of the medium such as refractive index, spatial distribution of occupied or unoccupied traps, etc. Second, the solitons rely solely on second order nonlinearities, $\chi^{(2)}$. Third, the self-trapping exists by virtue of the strong interaction and energy exchange between two or more beams at different frequencies as discussed in the first inset on optical nonlinearities. However, because of these constraints, quadratic solitons can only be launched in a limited class of materials, namely non-centrosymmetric media in which phase matching is possible, i.e. they only exist at reasonable powers over a narrow range of directions. Although it is now clear that quadratic solitons exist for any parametric mixing process involving $\chi^{(2)}$, and indeed they have been observed in optical parametric generators and amplifiers, to date they have been studied primarily during second harmonic generation.

One of the unique features of quadratic solitons is that they consist of *all* of the beams strongly coupled together by the second order nonlinearity: see Appendix 3 for a discussion of the physics of mutual trapping. For second harmonic generation, this means at least one fundamental field (one for Type 1 and two for Type 2 phase matching, respectively) and the harmonic field. Typically the harmonic field is narrower than the fundamental, as shown in Fig. 3. The relative composition of a quadratic soliton versus

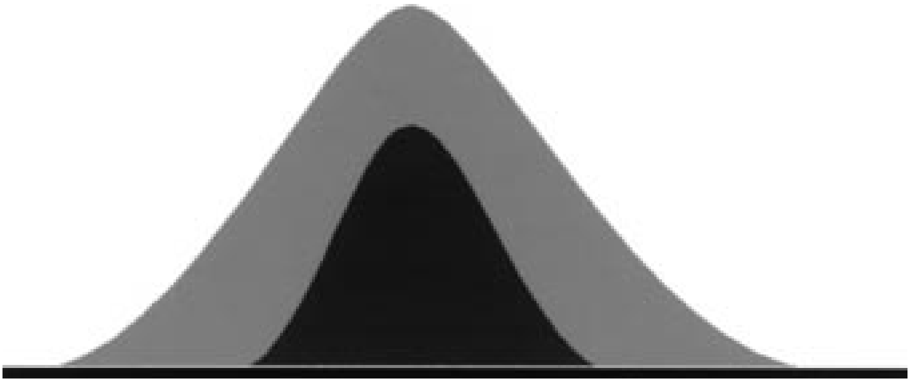

*Figure 3* Typical field distributions for a quadratic soliton with the fundamental being broader and the harmonic narrower (and darker).

detuning $(k_2 - 2k_1)L$ from the phase-matching condition is shown in Fig. 4 for the simplest Type 1, $(1 + 1)$ D case. Although this ratio depends somewhat on power, and the dimensionality of the medium, the general trend is that the harmonic content increases with decreasing detuning. In fact, as the detuning is increased for the $(1 + 1)$ D case and the harmonic content goes asymptotically to zero, the properties of quadratic solitons asymptotically approach those of Kerr solitons.

Despite their complexity, or maybe because of it, quadratic solitons behave as if their effective nonlinearity is saturable in nature. This can be argued as follows. First of all, the total electromagnetic energy is conserved so that if one beam increases power, it can only do so only at the expense of the other beam. This, in turn, weakens the effective self-trapping for at least one beam unless the beam size also decreases. However, this leads to stronger diffraction which again increases the beam sizes. Because of this multi-beam nature of the solitons, this self-regulating mechanism will not allow catastrophic self-focusing, independent of dimensionality, effectively behaving as a saturable nonlinearity.

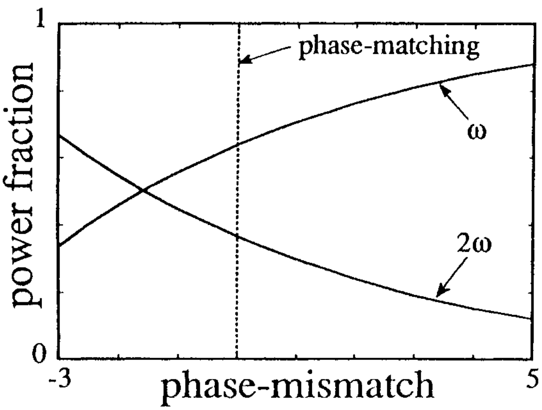


*Figure 4* The fraction of power carried by the fundamental ($\omega$) and second harmonic ($2\omega$) as a function of the detuning from phase-match for $(1 + 1)$D quadratic solitons (taken from L. Torner, *Opt. Commun.* **114** (1995) 136.

In contrast to photorefractive solitons where theory was followed immediately by experiment, quadratic solitons were first predicted back in the mid 1970s by Karamzin Sukhorukov [50], and 20 years passed before their stability was shown by Kivshar's group [51] and their existence was verified experimentally in (2 + 1) D by Torruellas *et al.* [52] and in (1 + 1) D by Schiek *et al.* [53] at CREOL. In the (2 + 1) D experiments, for example, a fundamental beam from a pulsed laser operating at 1064 nm was focused down to 20 μm at the input face of a standard bulk KTP doubling crystal that was 5 diffraction lengths long. Above a critical threshold intensity, both the output fundamental and harmonic beams collapsed from their diffracted beam sizes (observed at low input intensity) to diameters less than the fundamental input diameter, as shown in Fig. 5. Although the quadratic solitons consist of both the fundamental and harmonic beams, these experiments showed that the required second harmonic could be generated within the crystal and then the soliton formed.

Further experiments have established additional unique features of quadratic solitons. For example, for three wave mixing (two unique input beams), quadratic solitons have been found with a wide variety of relative composition for the three waves. For example, solitons supported by Type 2 phase matching involve two orthogonally-polarized fundamental beams and one second harmonic beam, as predicted by Buryak *et al.* [54] and Leo *et al.*, and recently demonstrated by Canva *et al.* Furthermore, the three interacting beams could also be of three different frequencies, as demonstrated experimentally earlier this year [55]. Another example is that of 'walk-off'. In harmonic generation it is not uncommon that the energy propagation directions (Poynting vectors) of the fundamental and harmonic beams are different. As long as the characteristic walk-off distance between the beams was less than the parametric gain length (see Appendix 3), the fundamental and harmonic beams were locked in space and co-propagated *together* as quadratic solitons [56–59].

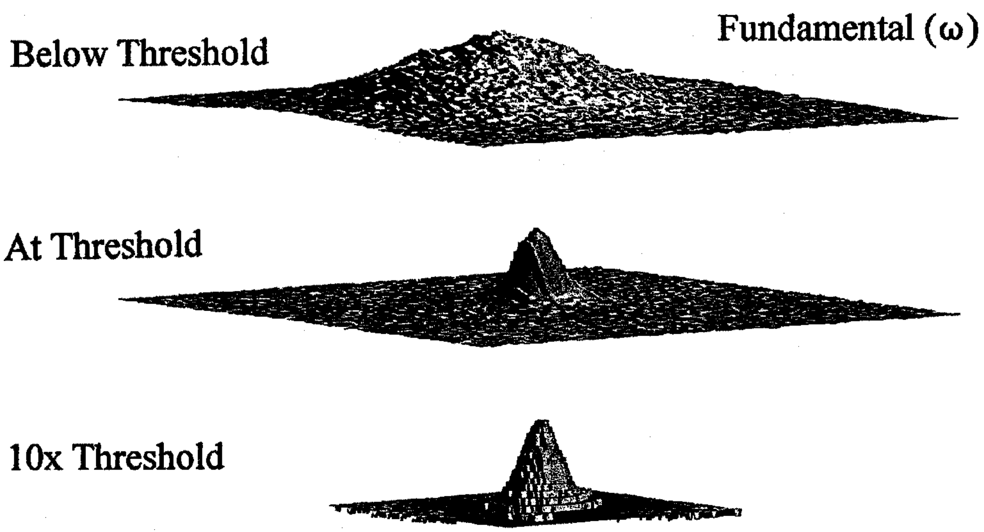


*Figure 5* (2 + 1) D quadratic solitons: intensity distributions of the fundamental beam ($\omega$) at the output plane at three different intensities. The harmonic ($2\omega$) beam looks the same. Note the collapse of the beam at intensities above the soliton-locking threshold. Figure taken from [52].

The area of quadratic solitons has suffered from a late blooming, since it took almost 20 years between the theoretical prediction and the first observation. Nonetheless, it is now rapidly advancing, in many interesting directions, including quadratic solitons in resonators [60], beam steering and control with quadratic solitona, [61, 62], reshaping [63] and transverse instabilities of $(1+1)$ D quadratic solitons propagating in a bulk medium [64–67].

## 6. Soliton interactions

Amongst all soliton properties, interactions (commonly referred to as 'collisions') between solitons are perhaps their most fascinating feature, since, in many aspects, solitons interact like particles. One way to intuitively understand soliton collisions is to consider the solitons in terms of their self-induced optical waveguides that are brought into close proximity. The solitons guided in these waveguides (in the form of guided modes) overlap, primarily in the 'centre' region between the waveguides where the evanescent 'tails' of the modes coexist. Now, there are two possible scenarios for these self-trapped beams to interact: *coherent* versus *incoherent* interactions. Both types of interactions are illustrated in Fig. 6.

### 6.1. Coherent interactions

Coherent interactions occur when the nonlinear medium can respond to interference effects between the overlapping beams. They occur in all nonlinearities with an instantaneous (or extremely fast) time response (such as the optical Kerr effect and the quadratic nonlinearity). For all other nonlinearities that have a fairly long response time (e.g. pho-

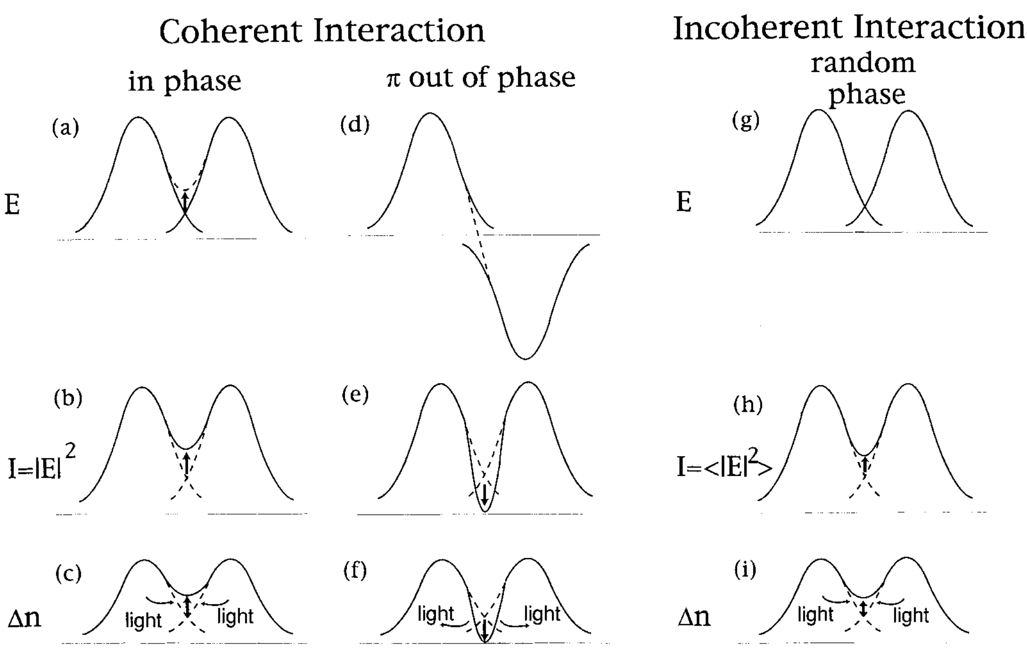


*Figure 6* Illustration of coherent and incoherent interactions between solitons.

torefractive and thermal), the relative phase between the interacting beams must be kept stationary on a time scale much longer than the response time of the medium. When this occurs, the material responds to interference between the overlapping beams. When the beams have a zero relative phase ('in-phase'), they interfere constructively and the intensity in the centre region between the induced waveguides is increased. In a self-focusing medium, this leads to an increase in the refractive index in that region, which in turn, attracts more light to the centre, moving the centroid of the solitons towards it and hence the solitons appear to attract each other. When the interacting beams are $\pi$ out of phase from each other, they interfere destructively and the index in the centre region is lower than it would have been if the beams were far away from each other. As a result, the solitons appear to repel each other. In fact, both 'attraction' and 'repulsion' between solitons are actually due to asymmetries in their induced waveguides that is caused (via the nonlinearity) by the close proximity of the beams.

### 6.2. Incoherent interactions

Incoherent interactions occur when the relative phase between the (soliton) beams varies much faster than the response time of the medium. In this case, the medium cannot respond to interference effects but responds only to the time-averaged intensity (average taken over a time longer than material response time), which is identical to a simple superposition of the intensities. Therefore, irrespective of their relative phase, the intensities of the beams add up and the intensity in the 'centre' region between the solitons is increased (as compared to a single isolated beam). Since these solitons propagate in a self-focusing medium, this leads to an increase in the refractive index in that region. As a result, more light is 'attracted' towards the centre region and the solitons appear to attract each other. Such an incoherent 'interaction force' is always attractive (for bright solitons), since the intensity in the centre region cannot decrease by merely the coexistence of two soliton beams at close proximity.

### 6.3. Collisions in Kerr media

The soliton interaction forces are, in principle, the same for all nonlinear media that can support solitons. There are, however, several very important differences in the outcome of collision processes between Kerr-type and saturable nonlinear media. First, in Kerr media all solitons are $(1+1)$ D and the collisions are bound to occur in one single plane. In addition, in Kerr media, all collisions are fully elastic, which implies that the number of solitons is always conserved. Furthermore, the system is integrable, and therefore no energy is lost (to radiation waves) but rather conserved in each soliton. In addition, the trajectories and 'propagation velocities' of the solitons recover to their initial values after each collision (whether attractive or repulsive). This equivalence between solitons and particles is the reason for the term 'soliton' [2]. In an attractive coherent collision in Kerr media in which the solitons' trajectories are separated by any non-zero angle, the solitons simply go through each other and remain virtually unaffected by the collision process (apart from a tiny displacement and a small change in absolute phase). When the attractive collision occurs at zero angular separation between the solitons, the solitons form a bound pair: they move towards each other, combine and separate periodically. On the other hand, in a repulsive Kerr collision the solitons simply move away from each other. Coherent collisions in Kerr media were demonstrated in [15–17].

## 6.4. Collisions in saturable nonlinear media

Collisions in saturable nonlinear media are, in many aspects, much richer than those in Kerr media and therefore more interesting. First, saturable nonlinear media can support $(2+1)$ D solitons and therefore collisions can occur in full three dimensions, giving rise to new effects that simply cannot exist in Kerr media. Second, as explained above, the self-induced waveguides in saturable nonlinear media can guide more than one mode. This gives rise to new phenomena, including soliton fusion, fission, and annihilation. In 1992 Gatz and Herrmann [68] found numerically that solitons in saturable nonlinear media which undergo a coherent collision at shallow relative angles can fuse to each other. One year later, Snyder and Sheppard [69] showed theoretically that colliding solitons can undergo 'fission', that is, generate additional soliton states upon collision, or, in other cases, annihilate each other. Their explanation was simple and elegant: since both solitons induce waveguides, one needs to compare the collision angle to the critical angle for guidance in these waveguide (that is, to the angle above which total internal reflection occurs and a beam is guided in the waveguide). In terms of a 'potential well', capture depends on whether the kinetic energy of the colliding wave-packets results in a velocity that is smaller than the escape velocity. If the collision occurs at an angle larger than the critical angle, the solitons simply go through each other unaffected (the beams refract twice while going through each other's induced waveguide but cannot couple light into it). If the collision occurs at 'shallow' angles, the beams can couple light into each other's induced waveguide. Now if the waveguide can guide only a single-mode (a single bound state), the collision outcome will be identical to that of a similar collision in Kerr media. However, if the waveguide can guide more than one mode, and if the collision is attractive, higher modes are excited in each waveguide and, in some cases, the waveguides merge and the solitons fuse to form one soliton beam. Such a fusion process is always followed by some (typically small) energy loss to radiation waves, much like inelastic collisions between real particles. It is important now to point out that colliding quadratic solitons exhibit effects similar to those of solitons in saturable nonlinearities, including soliton fusion [70–74]. The fact that soliton interactions/collisions, such as going through each other at steep angles, or fusion and fission at shallow angles, are similar to one another in all saturable nonlinear media, highlights the universality of soliton phenomena that are largely independent of the actual physical mechanism that enables them [75].

Experimentally, fusion of solitons was observed in all kinds of saturable nonlinear media: atomic vapour [76] photorefractives [77–82] and quadratic [83, 84]. Snyder and Sheppard [69] have also predicted that in a highly saturated regime, two colliding solitons may 'give birth' to a new soliton and three solitons could emerge after the collision (soliton 'fission'). A similar fission effect has recently been predicted to occur with quadratic solitons as well [74]. Indeed, this effect was recently observed in an elegant experiment by Krolikowski and Holmstrom [79] followed by a report on 'anihilation' of solitons upon collision [82]. These processes are illustrated by the experimental results of collisions between photorefractive solitons shown in Fig. 7. Figure 7a shows a top-view photograph of an attractive incoherent collision in which the solitons pass through each other at a large angle. An example of fusion in Fig. 7b shows the intensity distribution a long distance after a collision in which the same solitons collide at shallow angles and fuse to form, a single beam. (Figures 7a and b are taken from [77].) Similar results obtained with quadratic solitons [83, 84] are shown in Fig. 8 (taken from [83]).

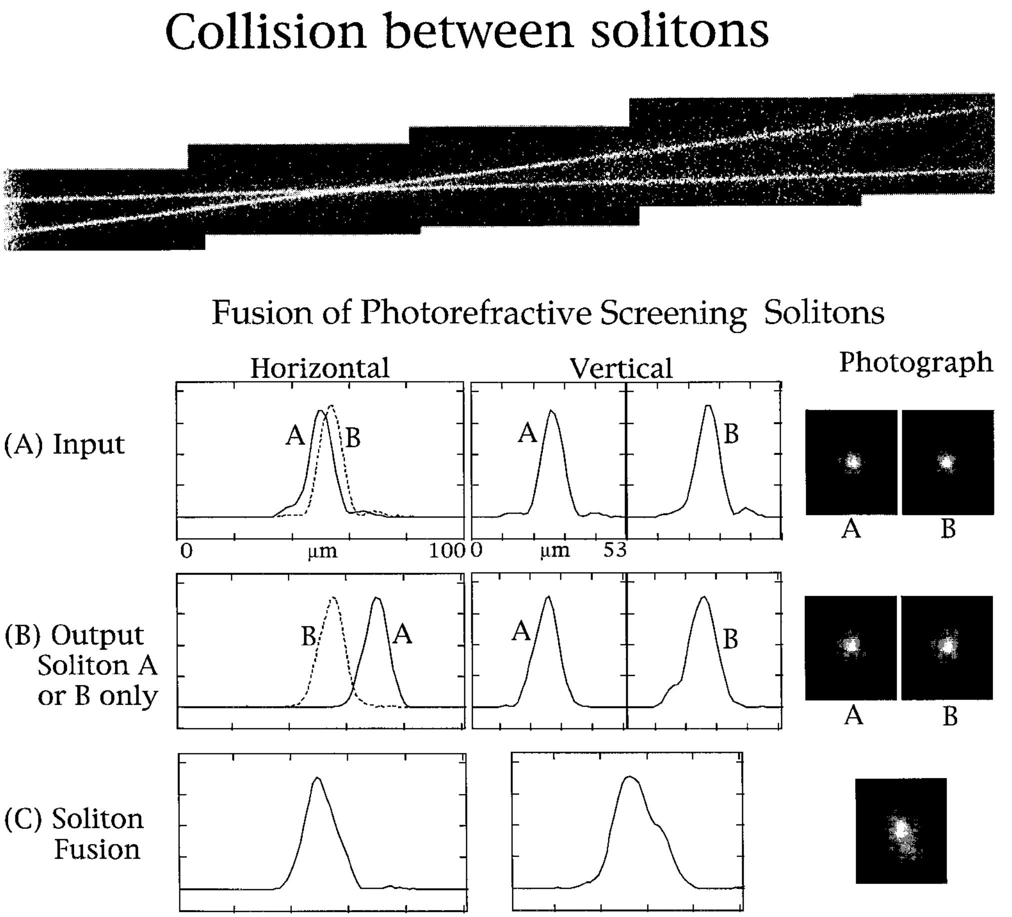


*Figure 7* (a) Top-view photograph of an (attractive) incoherent collision between two photorefractive screening solitonss in which the soliton pass through each at a large angle. (b) Fusion between the same solitons when the collision occurs at a shallow angle. Shown are the intensity profiles and photographs of beams A and B at (A) the entrance plane, (B) each individual soliton at the exit plane when the other is absent, and (C) the fused beam at the exit plane. Photographs taken from [77].

Soliton 'collisions' for phase differences intermediate between 0 and $\pi$ lead to energy exchange between the solitons. The interference pattern formed by the overlap of the 'tails' of the solitons is intermediate in phase to that of both of the solitons so that power is scattered from one soliton into the other. The higher intensity soliton narrows in space, and the weaker one broadens. An example of such a collision in $(1+1)$ dimensions for quadratic solitons is shown in Fig. 8 [83]. Note that the energy flow reverses in going from a phase difference of $\pi/2$ to $3\pi/2$. Similar effects have been seen in saturable Kerr media ($CS_2$) and photorefractive media [81].

Since saturable nonlinear media can support $(2+1)$ D solitons, one can also look at collisions of solitons with trajectories that do not form a single plane. When the solitons are individually launched, they move in their initial trajectories. When they are launched simultaneously, they interact (attract or repel each other) via the nonlinearity and their

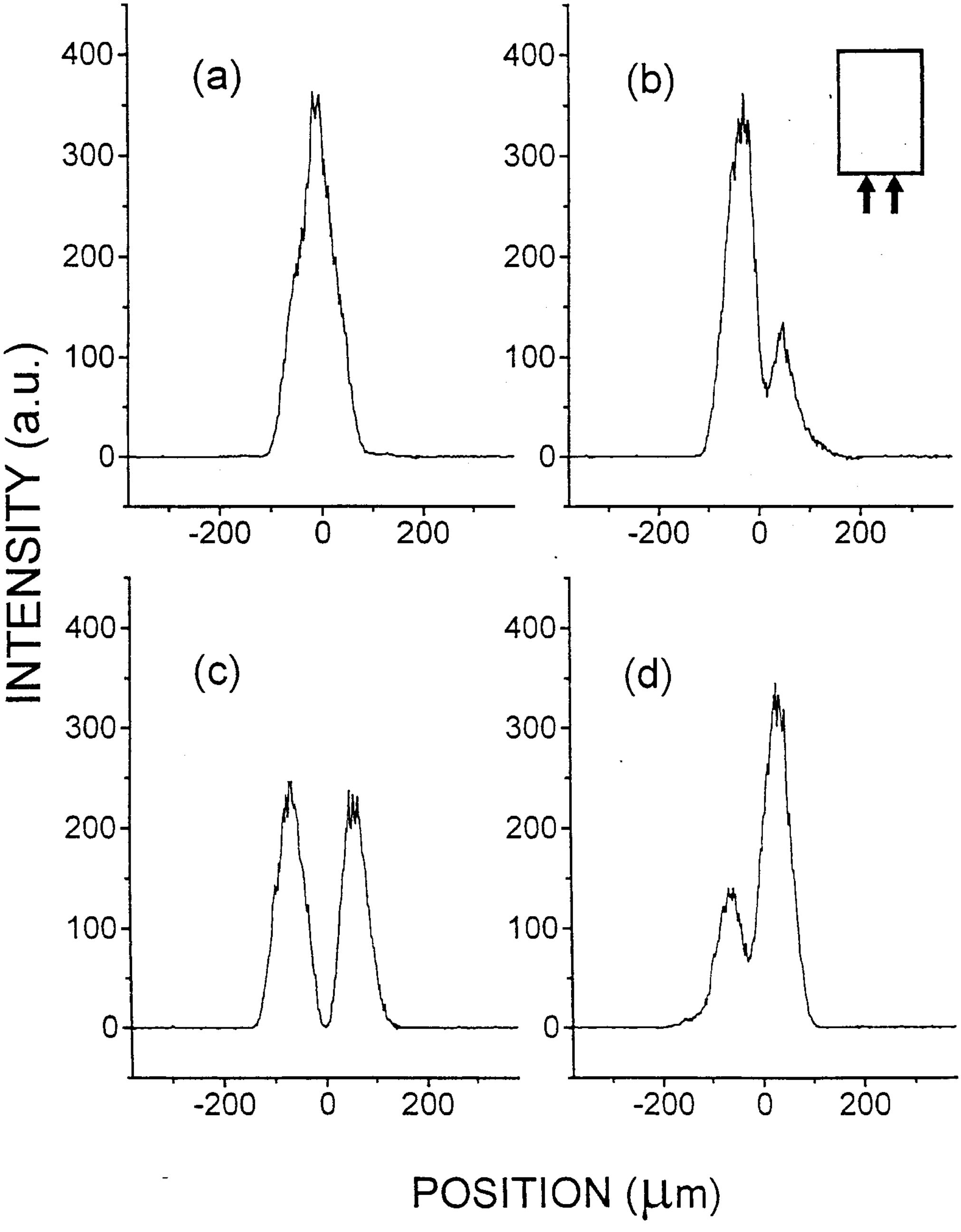


*Figure 8* The output from a collision between two (1+1) D quadratic solitons launched in parallel at the input. The relative phase angles for the four cases are (a) 0, (b) $\pi/2$, (c) $\pi$ and (d) $3\pi/2$. The second harmonic beam output looks the same. Figure taken from [83].

trajectories bend. If the soliton attraction exactly balances the 'centrifugal force' due to rotation, the solitons can 'capture' each other into orbit and spiral about each other, much like two celestial objects or two moving charged particles do. This idea was suggested by Mitchell *et al.* [85] in the context of coherent collision. Recently, Shih *et al.* [86] have demonstrated such spiraling–orbiting interaction employing an incoherent collision between photorefractive screening solitons. Under the proper initial conditions of separation and trajectories, the solitons capture each other into an elliptic orbit. This is shown in Fig. 9 (from [86]). If the initial distance between the solitons is increased, the solitons' trajectories slightly bend toward each other but their 'velocity' is larger than the escape velocity and they do not form a 'bound pair'. On the other hand, if their separation is too small, they spiral on a 'converging orbit' and eventually fuse. It is important to note that

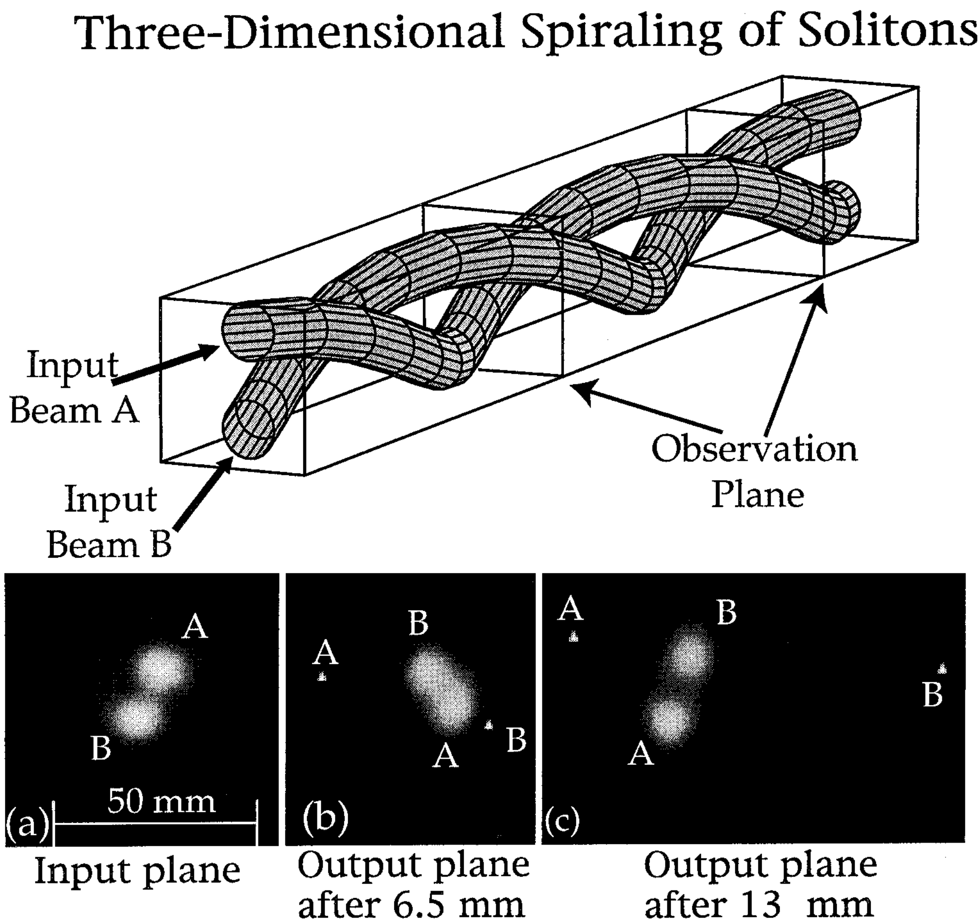


*Figure 9* Spiraling of two colliding photorefractive screening solitons with initial trajectories that do not lie in the same plane. Shown are the photographs of the optical beams. (a) Beams A and B at the input plane, (b) the spiraling soliton pair afer 6.5 mm of propagation and (c) the spiraling pair after 13 mm of propagation. Note that spiraling occurs in elliptical orbits. The triangles indicate the centre of the corresponding diffracting beams. After 6.5 mm the solitons have spiraled about each other by 270° after 13 mm the spiraling angle doubles to 540°. Photographs taken from [86].

spiraling-fusion and spiraling-repulsion of solitons was first observed by Tikhonenko *et al.* [76, 87] who have generated the solitons from the breakup of a vortex beam in a saturable self-focusing medium. However, because in this case the interaction is coherent, the solitons were never able to attain stable orbits of spiraling about each other, and always either fused to form a single beam [76], or 'ran away' from each other [87]. Nonetheless, that was the first observation of full 3D interaction of optical solitons of any kind.

The observation of spiraling brings about an interesting question: *Do interacting spatial solitons also conserve angular momentum?* Recent calculations by several groups seek universal answers to this question. Recently, 3D spiraling was also predicted for quadratic solitons [88], thereby indicating that this kind of interaction is general, and should exist in many 3D nonlinear systems of self-trapped wave-packets in nature.

## 7. Dark and vortex solitons

Having shown that optical beams can self-trap in a nonlinear medium, a logical question is: *can darkness be self-trapped as well?* Or, in more general form, can a 'null' borne on a uniform wave be self-trapped? The answer to that is positive, but it requires a nonlinearity of a sign opposite to that which supports bright solitons. In other words, a dark 'notch' (or pulse) can indeed self-trap, but it requires a self-defocusing nonlinearity. In Kerr media, it implies that the sign of $n_2$ must be negative, i.e. $\Delta n = n_2|E|^2 < 0$. Stable dark solitons also exist in saturable media and their properties are very similar to those in Kerr media.

Intuitively, one can understand dark solitons in the following manner. A narrow dark notch (stripe) borne on an otherwise-uniform light beam propagates in a nonlinear material. If the nonlinearity is zero (the medium is linear), the notch diffracts (broadens) much like a narrow beam does. When the notch-bearing beam propagates in a self-defocusing medium, as a result of the illumination the refractive index decreases in the illuminated region (on both sides of the dark notch), whereas at the centre of the notch the index remains unchanged. Because the index at the centre is now higher than in the illuminated regions, the two portions of the beam (on either side of the notch) expand their inner boundaries and reduce the diffraction of the notch. Under specific conditions the diffraction of the notch is fully eliminated and the dark notch experiences stationary (non-diffracting) propagation as a dark soliton. It is worth noting that a dark soliton not only confines the dark notch, but also induces an effective graded-index waveguide that can guide other beams in the centre of the notch (since the refractive index there is higher than that outside the notch). It turns out that fundamental dark solitons must possess an anti-symmetric waveform, i.e. their amplitude must undergo a $\pi$ phase jump in the centre of the notch. On the other hand, if the phase front of the notch-bearing beam is uniform (an 'amplitude notch'), the notch evolves into two separate dark or grey (minimum intensity does not go to zero) solitons that diverge away from each other (commonly referred to as a Y-junction soliton). If the self-defocusing nonlinearity is large and the input notch is fairly broad, an anti-symmetric input notch can evolve into any odd number of dark or grey solitons, all diverging away from each other. On the other hand, an amplitude notch can evolve into any even number of dark solitons which also diverge away from each other.

Like their bright counterparts, dark solitons were predicted by Zakharov and Shabat [89]. The first experimental observation of dark solitons was carried out by Kaplan's group (both in sodium vapour and in a slightly absorbing liquid that possessed a thermal nonlinearity) [90, 91] and in the same year by Smirl's group in bulk ZnSe [92]. Because dark solitons induce an effective waveguide structure (the index in the centre of the notch is higher than that at the margins), they can be used to guide and steer other beams. Indeed, this has been demonstrated in a series of papers by Luther-Davies group from Australia [93, 94] and followed later on by Stegeman's group demonstrating all-optical steering of arrays of a dark soliton [95]. Following the discovery of photorefractive solitons, dark photoreactive solitons were observed in the form of quasi-steady-state solitons [96], photovoltaic solitons [97, 98] and screening solitons [37, 99–101].

A natural question to ask now is: *'can a dark hole self-trap?'* or: *'is it possible to generate a (2 + 1) dark soliton?'*. This question has been answered for a Kerr self-defocusing nonlinearity in superfluidity, by Ginzburg and Pitaevskii [102], and more recently for an optically nonlinear medium [103]. The two-dimensional equivalent of dark solitons (whose phase jumps by $\pi$ at the centre the notch), is a self-trapped beam with a phase front in the form of $\exp[\mathrm{i}m\theta]$, where $\theta$ is the azimuthal angle and $m$ an integer referred to as the 'topological charge'. The intensity pattern of such a beam consists of a dark filament borne on uniform beam. For $m = 1$, the phase along every straight line that goes through the 'origin' (centre of the beam) undergoes a $\pi$ phase jump at the origin, irrespective of $\theta$. An elegant way to visualize this phase front is to interfere such a vortex soliton with a spherical wave. The resultant interference pattern consists of a vortex that starts at the origin. Such a self-trapped vortex is illustrated in Fig. 10, which shows the experimental results with a vortex soliton supported in the photovoltaic nonlinearity in a bulk $LiNbO_3$ crystal (taken from [104]). Experiments with optical vortex solitons were first reported by Swartzlander and Law [105, 106] in 1992 in liquids exhibiting a Kerr-like thermal nonlinearity. These were followed by experiments in saturable nonlinearities, such as rubidium vapour [107–109] and elegant demonstrations of interactions between vortex solitons [107] (nonlinear rotation about each other). In photorefractive media, singly-charged vortex solitons were demonstrated with quasi-steady-state solitons [96], photovoltaic solitons [104] and screening solitons [110]. With respect to higher-order vortices, Mamaev *et al.* [111] have shown that the anisotropic nature of the photorefractive nonlinearity can, in some cases (when the applied field is too high), lead to disintegration of a higher-order vortex of order $m$ into $m$ unit-charge isolated vortices. They have also shown that when a dark stripe is launched into a bulk (3D) photorefractive medium and the nonlinearity is much higher than that needed to support a $(1+1)$ D dark soliton, the stripe breaks up into multiple vortex solitons ('snake instability') [112].

In summary of this section, it is apparent that dark and vortex spatial solitons which were dormant for almost 20 years (in the absence of experimental observations), have become a very important research topic (for an excellent recent review on dark solitons, see [113]). Many new features of dark solitons have been predicted recently. For example, quadratic dark solitons have been predicted by Buryak and Kivshar [114]. Another idea is the breakup of a vortex beam into multiple bright solitons, which has been demonstrated recently in saturable self-focusing media [76] and in photorefractive media [110], is now predicted for quadratic media [115, 116] and are being studied in the context of conservation of angular momentum [117]. Once again, there are striking similarities among the properties of dark and vortex solitons in all nonlinear media.

## 8. Vector and two-component solitons

In the previous sections, self-trapping of *scalar* optical beams, that is, beam propagation and self-trapping governed by a single equation, was discussed. Electromagnetic waves, however, have two possible (orthogonal) polarization states. When a general (elliptically-

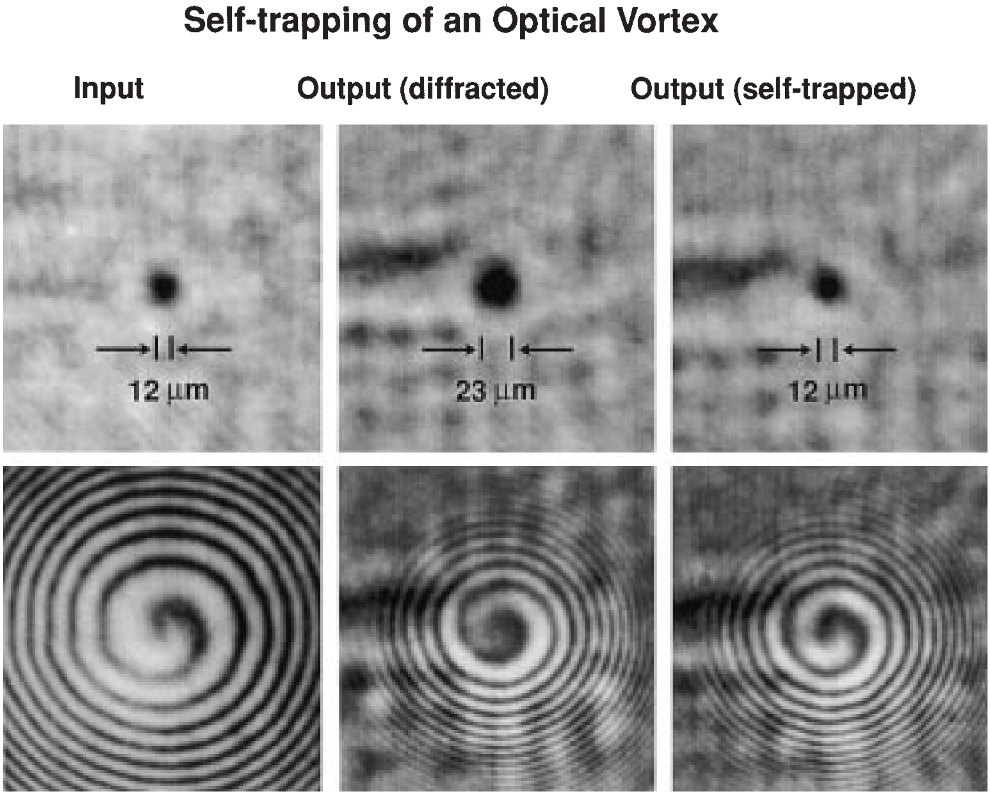


*Figure 10* Self-trapping of a singly-charged vortex beam in a photovoltaic-photorefractive crystal. Top: intensity profiles of the vortex-nesting beams. Bottom: interference of the vortex beam with a spherical wave. Left: At the input plane. Middle: At the output plane with the non-linearity absent (normal diffraction). Right: The self-trapped vortex. Photographs taken from [104].

polarized) optical beam propagates in isotropic media, each field component is fully decoupled from the other. But, when this medium is nonlinear, this beam can self-trap into a *two-component (vector) soliton*. This was first suggested by Manakov [118], who has shown that vector solitons can indeed exist provided that three conditions on Kerr nonlinearities are satisfied. (i) The self-phase modulation in the medium is identical for both polarization components (i.e. $n_2$ is the same for both polarizations). (ii) That the self-phase-modulation is identical to the cross-phase modulation (i.e. the effect of each component on itself is identical to its effect on the other component). (iii) That the coherent wave mixing term, which couples the two polarizations and can lead to power exchange between them, is absent, i.e. the polarization components are coupled only through their intensities only. These requirements are rather unusual because there is no crystal symmetry for which (ii) is *a priori* satisfied and because the coherent mixing term can only be removed if the beams are incoherent with respect to each other. For example, in a general isotropic medium the ratio between self- and cross-phase modulation is 3/2 and the coherent term is not zero. However, it turns out that these conditions can be more-or-less satisfied using Kerr nonlinearities in AlGaAs quantum well structure at particular optical wavelengths when the coherence between the two polarizations is destroyed, and last year

Kang *et al.* [119] observed these solitons experimentally. These authors were also able to demonstrate vector solitons in which power is continuously exchanged between the two polarization components with propagation distance, i.e. the two orthogonally polarized beams are coherent (versus the Manakov case in which the coherence was removed) [120].

At this point it is important to note that all quadratic solitons are, strictly speaking, vector solitons, as they always involve at least two components of the electromagnetic fields of two different frequencies and can also be of different polarizations (and in fact typically are so, if birefringence is used for phase matching).

Both Manakov solitons and quadratic solitons observed thus far exhibit a single intensity peak for either one of the vector components. This means that in terms of the self-consistency principle, each component occupies only the lowest mode in the effective waveguide by the total intensity of the beam. However, solitons in saturable nonlinear media can induce *multi-mode waveguides* [25]. This raises the interesting possibility of having more than one guided mode populated (in contradistinction with fundamental solitons in such media, which form the lowest mode and leave the other guided mode unpopulated). This extends the ideas of two-component solitons to bi-modal beams, that is, beams that are jointly self-trapped but consist of two *different* guided modes of their jointly-induced waveguide. The total intensity distribution of such a two-component soliton can be 'double-humped'. However, for such a composite soliton to exhibit a stationary profile throughout propagation, the modes must *not* interfere with each other, since such interference would cause a periodic oscillation in the optical intensity and thus give rise to a periodic induced waveguide [121]. Elimination of interference between modes can be achieved in two ways: (a) the modes being polarized orthogonal to one another, which gives rise to the Manakov solitons discussed above, or (b) the modes being incoherent with respect to each other, that is, the relative phase between the mode varies in time much faster than the response time of the nonlinear medium, which *must be non-instantaneous*. When the relative phase between the modal components varies in time much faster than the material can respond, the components couple to each other (via the nonlinearity) through their intensity only (because all index changes driven by interference effects are averaged out). Under some conditions, both components self trap in their jointly induced waveguide. This idea was first proposed by Christodoulides *et al.* [122] in the context of photorefractive solitons, and demonstrated last year at Princeton [123]. However, in all of these studies both soliton components have populated the lowest guided mode in their jointly induced waveguide only.

An intermediate step towards generating a multi-mode soliton was a bright–dark soliton pair. This is a bi-component beam in which one component is 'bright' and the other 'dark', together forming a bright–dark coupled pair [124–126]. Bright–dark coupled soliton pairs were first demonstrated using two different wavelengths instead of two orthogonal polarizations for the different components [127], and more recently with incoherently-coupled bright-dark photorefractive soliton pairs [128, 129]. With the new understanding that a dark soliton is actually the second guided-mode of its self-induced waveguide at the cutoff frequency [130] these bright–dark soliton coupled pairs are now conceived as an intermediate stage for obtaining truly multi-mode solitons.

The next step towards proposing multi-mode solitons were degenerate temporal vector solitons, which are bi-modal with respect to their common induced potential. These were identified in birefringent optical fibres [131, 132]. Finally, true multi-mode solitons were predicted first for temporal solitons in isotropic Kerr media [133] and shortly thereafter for spatial solitons [134] in a medium with a saturable self-focusing nonlinearity, for which the total intensity of the beam induces a multi-mode waveguide. The latter has shown that bi-modal spatial solitons can exist even when the polarization state of each component is dynamically varying throughout propagation (so-called '*dynamic solitons*'), provided that the polarization states of the components remain orthogonal to each other everywhere. In this case the jointly induced waveguide is *stationary* (does not change during propagation). In general, multi-mode solitons making use of orthogonal polarizations have a limiting feature: because only two polarization states are possible, the maximum number of components (modes) involved is *two*. Using the second approach (mutual incoherence) to create multi-mode solitons, allows having *more than two modes* in the soliton. In some cases *all* of the guided modes of a jointly induced waveguide can be populated. Such multi-mode multi-hump solitons have recently been observed at Princeton [135]. The multi-mode solitons were made by two different populated modes of their jointly self-induced wave-guide with the same polarization. Double- and triple-humped beam profiles were observed corresponding to combinations of the first + second modes, and second + third modes, respectively. The observation of multi-humped solutions is due to the fact that the higher modes are sufficiently populated to see the structural features of the higher modes. With this in hand, the road towards demonstrating multi-component spatial solitons consisting of three, four, or more components, is now paved. Furthermore, the concept of multi-component solitons is already spreading towards quadratic solitons: first prediction of bright–dark 'simultons' was made by Trillo [136] in 1996, and was followed by a prediction of multi-humped quadratic solitons made by Haelterman *et al.*[137]. In materials for which the second order nonlinearity is close to resonance (e.g. intersubband transitions in quantum well structures), it is conceivable that one can take advantage of the non-instantaneous response and generate multi-component quadratic solitons. Finally, as will be discussed in the next section, multi-component solitons are closely related to incoherent solitons.

## 9. Incoherent solitons: self-trapping of incoherent wave-packets

As the final section of this review on solitons, I wish to introduce a new, perhaps somewhat futuristic, direction into which solitons seem to be evolving: *incoherent solitons*.

Until 1995, all soliton experiments employed a coherent 'pulse', either in space, time, or both. In other words, given the phase at a given location on the pulse (space or time) one can predict the phase anywhere on that self-trapped pulse. However, pulses or wave-packets do not necessarily need be coherent. For example, one can focus into a narrow spot a light beam from a natural source such as the sun or an incandescent light bulb. Can such beam self-trap in a nonlinear medium?

In 1996, Mitchell *et al.* [138] at Princeton demonstrated self-trapping of beams (spatial 'pulses') upon which the phase varied randomly in time/space across any plane. In the first experiment, a quasi-monochromatic partially spatially-incoherent light beam was employed. The beam originated from a laser and passed through a rotating diffuser that introduced a new (random) phase pattern every 1 μs. The beam was launched into a slowly-responding photorefractive crystal and, under appropriate conditions, the envelope of this beam self-trapped into a single non-diffracting narrow filament. Last year, in a subsequent experiment Mitchell and Segev [139] demonstrated that an *incoherent white light beam*, i.e. a 'pulse' that is both temporally and spatially incoherent, *self-traps*. In this experiment the self-trapped beam originated from a simple incandescent light bulb which emitted light between 380–720 nm wavelength (see Fig. 11). In yet another experiment, self-trapping of dark incoherent 'beams', i.e. 1D or 2D 'voids' nested in a spatially incoherent beam, was also demonstrated [140].

To understand the new ideas involved, some aspects of incoherent light have to be explained first. A spatially-incoherent beam is nothing but a multi-mode (so-called 'speckled' beam) whose structure varies randomly with time. The beam consists of bright and dark 'patches' (thus the notion 'multi-mode') that are caused by a random phase distribution, which varies randomly with time. The envelope of this beam is defined by the time-averaged intensity. To illustrate this, consider a detector array (e.g. human eye) which monitors the beam. When this detector responds much more slowly than the characteristic phase fluctuation time, all it will 'see' is the time-averaged envelope. Typically, such an incoherent beam diffracts much more than a coherent beam, since every small bright 'patch' (speckle) contributes to the diffraction of the envelope. In the limiting case of the speckles being much smaller than the beam size, diffraction is dominated by the degree of coherence, i.e. the size of the speckle, rather than the diameter of the beam's envelope.

Instantaneous nonlinearities *cannot* self-trap such a beam. If a spatially incoherent beam is launched into a self-focusing nonlinear medium that responds instantaneously (e.g. the optical Kerr effect), each small speckle forms a small 'positive lens' and captures a small fraction of the beam. These bright–dark features on the beam change very fast throughout propagation and these tiny induced waveguides intersect and cross each other in a random manner. The net effect is beam breakup into small fragments and self-trapping of the beam's envelope will not occur.

For self-trapping of an incoherent beam (an incoherent soliton) to exist, several conditions must be satisfied. First, the nonlinearity must be non-instantaneous with a

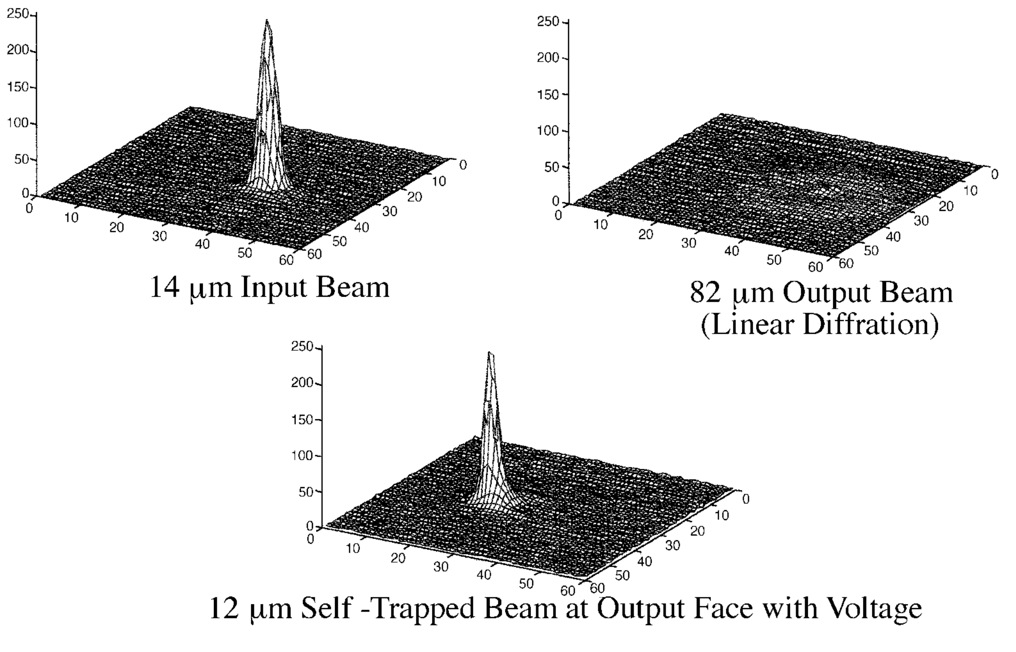


*Figure 11* Self-trapping of an incoherent white light beam. Figure taken from [139].

response time that is much longer than the phase fluctuation time across the incoherent beam. Such a nonlinearity responds to the time-averaged envelope and not to the instantaneous 'speckles' that constitute the incoherent beam. Second, the multi-mode (speckled) beam should be able to induce a multi-mode waveguide via the nonlinearity. Otherwise, if the induced waveguide is able to support only a single guided mode, the incoherent beam will simply undergo spatial filtering, radiating all of its power but the small fraction that coincides with that guided mode. Third, as with all solitons, self-trapping requires self-consistency: the multi-mode beam must be able to guide itself in its own induced waveguide. Theory of incoherent solitons was presented in several recent papers. The first theoretical approach analysing self-focusing of incoherent beams in non-instantaneous nonlinear media was suggested by Christodoulides *et al.* [141]. It is based on a coherent density model†, as a nonlinear-optics extension to the van Cittert–Zernike theorem typically used in linear optics. Its strength lies in its ability to analyse dynamical propagation behaviour of incoherent beams, but it cannot find the incoherent soliton solutions. Then, Mitchell *et al.* [145] have suggested a modal theory which is based on the three principles described above. With this approach, the soliton solutions are found, along with their properties (shape, coherence function) and the range in parameter space that supports them. This modal theory was also used later on to describe dark incoherent solitons [146]. In fact, it has recently been shown that the coherent density approach and the modal theory of incoherent solitons are fully equivalent to one another [147]. From both theories it is apparent that the self-trapping process re-shapes the statistics of the incoherent beam. For example, incoherent sources (e.g. the Sun) have non-localized statistics: the correlation length (loosely defined as the distance beyond which two points are no longer phase correlated) does not depend on the absolute location. In the incoherent soliton, however, the correlation length has a different value at the centre of the beam and at its margins. Furthermore, it is possible to 'engineer' (at least to some extent) the coherence properties of an incoherent beam by the self-trapping process [148]. Finally, a recent paper by Snyder and Mitchell [149] employs a geometrical optics approach for analysing bright incoherent solitons in the extreme limit of beams that are much larger than their correlation distance. It is useful primarily to provide qualitative results on the ability to trap 'big' incoherent beams in a given nonlinear medium.

The rapid progress in this new area of incoherent solitons brings about many interesting fundamental ideas (such as coherence control) and possible applications (e.g. using self-trapped beams from incoherent sources, such as, light emitting diodes) for reconfigurable optical interconnects and beam steering. It is a very young area, nonetheless, it seems to have related aspects in many other areas of physics, in which both nor-linearities and statistical (ensemble) averaging are involved.

## 10. Summary

This article is an attempt to convey the rapid progress, the excitement and the new physics which is emerging from investigations of optical spatial solitons. Although the propagation distances involved are certainly not on the scale of the temporal solitons in optical

† The coherent density approach is related to a pioneering suggestion made by Hasegawa [142, 143] to generate incoherent solitons in plasmas. Later on, Hasegawa also suggested *incoherent temporal solitons* in multi-mode optical fibres [144]. Thus far, neither incoherent plasma solitons nor incoherent temporal solitons in multi-mode fibres was observed experimentally.

fibres (which have pioneered solitons in optics), the variety of nonlinearities accessible is far broader and the physical phenomena much richer. One can expect that this effort will lead to a deeper understanding of nonlinear dynamics, especially in view of the large and continuously increasing number of features that have been identified to be common to all solitons in nature.†

In writing this paper, I have used the basic ideas laid out in a *Physics Today* article I co-authored with George Stegeman of CREOL, on *self-trapping of optical beams*. In fact, this article is based on the first draft of that paper, which was far too long for *Physics Today*. *De facto*, George Stegeman is a 'hidden co-author' of my article. I appreciate his help and friendship and the many insights he has provided. I would like to dedicate this review paper to Bruno Crosignani and Greg Salamo and to my post-doctoral fellows and graduate students who have made the dream on photorefractive solitons and on incoherent solitons come true: Ming-feng Shih, Zhigang Chen and Matt Mitchell. I also wish to thank my friends and collaborators who have gone with me a long way over the last six years: Demetri Christodoulides, Yuri Kivshar, George Valley, Paolo DiPorto, Marty Fejer, Matt Bashaw, Minoru Taya, Galen Duree, Mark Garrett, Toni Degasperis, Stefano Trillo, Matthew Chauvet and Amnon Yariv.

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† 'Solitons are a gift from God. Therefore it is a sin not to use them'. A. Hasegawa at a Colloquium at Princeton University, December 1997.

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## Appendix 1: optical nonlinearities

All optical spatial solitons rely on some form of optical nonlinearity to facilitate beam trapping. Here I introduce a simple model for nonlinearities for the non-expert.

I draw on the simple harmonic oscillator, a one-dimensional spring with an electron on the end in which the optical polarization induced is proportional to the spring displacement $x(t)$. When the spring–electron system is excited at the applied optical frequency, the restoring force, $k_0x(t)$, is proportional to the oscillation amplitude $x(t)$ with a proportionality coefficient (the spring constant) $k_0$. At large enough amplitudes, however (assuming the spring does not break), the displacement saturates. When the amplitude is not extremely large, one can expand the restoring force as a power series in $x(t)$ and obtain an effective spring coefficient $k[x] = k_0 + k_1x - k_2x^2 - k_3x^3 + k_4x^4 - \dots$ (for the expansion to converge, of course, the terms in the series must be of a decreasing magnitude). Taking for now just the even power terms in the displacement, $k[x] = k_0 - k_2x^2 + k_4x^4$ indicates a weak *symmetric* anharmonicity. The motion of the electron leads to a material polarization which has terms linear, cubic, quintic etc. in the applied optical fields and leads to odd order nonlinearities $\chi^{(3)}, \chi^{(5)}$ etc. From all of the possible terms cubic in the field, take $\mathbf{P}_{\mathrm{NL}} = \epsilon_0 \chi^{(3)} |\mathbf{E}|^2 \mathbf{E}$ as the pertinent nonlinear polarization for self-trapping and the equation which describes the evolution of the optical field is nothing but the nonlinear Schrodinger (NLS) equation with a cubic potential. In other words, the refractive index, $n$, can be written as $n = n_0 + n_2|E|^2$, where $n_0$ is the background refractive index, $E(t,t)$ is the electric field amplitude of the EM wave, and $n_2$ is some coefficient whose sign depends on the actual anharmonicity. For this approximation to be true the optically-induced change in the refractive index, $n_2|E|^2$ must be much smaller than $n_0$. The nonlinear effect involving $n_2|E|^2$ is called the optical Kerr effect. In the same spirit, one can also add the next term in the expansion of the symmetric anharmonic spring, $k_4 x^4$, which leads to a quintic optical nonlinearity in the form of $n_4|E|^4$ (note that if $n_2$ and $n_4$ have opposite signs, then $n_4|E|^4$ tends to saturate the index change). In more complicated material systems (such as solids) the frequency dependence of the electric susceptibility is different, nonetheless the weak anharmonicity concept that gives rise to the Kerr nonlinearity remains the same.

The even order nonlinearities (for example, the second order one $\chi^{(2)}$) derive from a weak *asymmetric* anharmonicity with respect to the electron displacement, i.e. the relevant anharmonic terms in the effective spring coefficient are $k[x] = k_1 x + k_3 x^3 \ldots$. The leading term leads to an induced nonlinear polarization $\mathbf{P}_{\mathrm{NL}} = \epsilon_0 \chi^{(2)} \mathbf{E}_1 \mathbf{E}_2$ due to the mixing of two fields which is possible only in non-centrosymmetric media. It leads to three-wave-mixing processes such as the electro-optic effect and frequency conversion. Through the electro-optic effect, an externally-applied DC electric field (say, $\mathbf{E}_2 = \mathbf{E}_0$) produces a polarization at the optical frequency (that of $\mathbf{E}_1$) and modifies the index of refraction in the medium as $\Delta n = (-n^3/2)\tilde{\mathbf{r}} \cdot \mathbf{E}_0$, where $\tilde{\mathbf{r}} \propto \chi^{(2)}$ is the third rank electro-optic tensor that reflects the crystalline symmetry. The electro-optic effect is the very same effect used in optical modulators in optical communication links (fibres). As explained in the text, in photorefractive materials it also leads to photorefractive solitons. For the mixing of two fields of optical frequencies $\omega_1$ and $\omega_2$, a nonlinear polarization of the form $\mathbf{P}_{\mathrm{NL}}(\mathbf{r},t) \propto \chi^{(2)} \mathbf{E}_1(\mathbf{r},t) \mathbf{E}_2(\mathbf{r},t)$ is formed at the sum $(\omega_1 + \omega_2)$ and difference $(\omega_1 - \omega_2)$ frequencies. This effect is the 'heart' of frequency conversion in nonlinear optics and is commonly used for second-harmonic generation, parametric oscillation, etc. As explained in the text, it also leads to quadratic solitons. For these second-order frequency conversion processes, one needs to keep in mind that strong coupling and energy exchange between the waves requires *momentum conservation* between the two input and one output photons. In optics this is referred to as wavevector conservation or *phase matching*, the condition for which the interacting beams travel at the same (or almost the same) phase velocity. This wave-vector-matching condition between the waves imposes the strongest constraints on efficient frequency conversion (and hence on the observation of quadratic solitons).

## Appendix 2: equations governing single-component spatial solitons

In a general optically nonlinear medium, the equation that governs the propagation of a single quasi-monochromatic optical beam is

$$\frac{\partial}{\partial z} a(x,y,z) - \frac{\mathrm{i}}{2k}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) a(x,y,z) = \frac{\mathrm{i}k}{n_0} \Delta n(|a|^2) a(x,y,z)$$

where the electric field of the optical beam is $E(\mathbf{r},t) = 0.5\,a(x,y,z)\,\exp[\mathrm{i}(\omega t - kz)] + \mathrm{c.c.}$ .

For the particular case of $(1+1)$ D bright Kerr-type solitons, $\Delta n(|a|^2) = n_2|a|^2 > 0$ and $\partial a/\partial y = 0$, so the equation of propagation reduces to

$$\left[\frac{\partial}{\partial z} - \frac{\mathrm{i}}{2k}\frac{\partial^2}{\partial x^2}\right] a(x,z) = \frac{\mathrm{i}k}{n_0} n_2 |a(x,z)|^2 a(x,z)$$

whose simplest solitary (stationary) solution is of the form $a(x,z) = u(x)\,\exp[\mathrm{i}\gamma z]$ ($\gamma$ real), where $u(x) = u_0\,\mathrm{sech}[\alpha x]$ with $\alpha = k u_0 (n_2/n_0)^{1/2}$ and $\gamma = \alpha^2/(2k) = k n_2 u_0^2/(2n_0)$, which is also the fundamental Kerr-type soliton. In saturable nonlinear media, $\Delta n(|a|^2)$ can attain the forms described in the text in the sections on saturable nonlinearities and on photorefractive solitons.

## Appendix 3: self-trapping of quadratic solitons

Quadratic solitons consist of beams at two or more frequencies which are strongly coupled by second order nonlinearities under conditions of wavevector conservation. Here I discuss the self-trapping mechanism in physical terms for the simplest case of Type 1 (a single input fundamental beam) second harmonic generation in a $(1+1)$D slab waveguide geometry. To do this I borrow from standard textbooks on nonlinear optics the coupled mode equations which describe the wavevector-matched interaction between a funamental ($E_1(\mathbf{r},t) = 0.5\,a_1(y,z)\,\exp\{\mathrm{i}[\omega t - k_1 z]\} + \mathrm{c.c.}$) and second harmonic ($E_2(\mathbf{r},t) = 0.5\,a_2(y,z)\,\exp\{\mathrm{i}[2\omega t - k_2 z]\} + \mathrm{c.c.}$) beam. In the presence of diffraction along the $y$-axis,

$$-2\mathrm{i}k_1 \frac{\partial}{\partial z} a_1(y,z) + \frac{\partial^2}{\partial y^2} a_1(y,z) = -\Gamma a_1^*(y,z) a_2(y,z)$$

$$-2\mathrm{i}k_2 \frac{\partial}{\partial z} a_2(y,z) + \frac{\partial^2}{\partial y^2} a_2(y,z) = -\Gamma a_1^2(y,z)$$

where $\Gamma$, the nonlinear coupling coefficient, is proportional to $\chi^{(2)}$. The first, left-hand side terms describe the change in the complex amplitude of each beam due to diffraction (second left-hand side terms with double derivatives) and the source terms on the right-hand side. The key to self-trapping is the structure of the source terms which consist of the product of the two fields of finite spatial extent. Consider first the generation of the second harmonic which is driven by the term $a_1^2(y,z)$. The generated second harmonic is therefore initially narrower along the $y$-axis than the fundamental. The characteristic distance over which significant power transfer to the harmonic occurs is called the parametric gain length $L_{\mathrm{PG}} = [\Gamma a_1(0,0)]^{-1}$. The fundamental beam, $a_1$, is regenerated via the product $a_1^*(y,z)\,a_2(y,z)$ and, if the second harmonic field is narrower than the fundamental, this regenerated field is also narrower than the fundamental field unconverted to the harmonic. Therefore, both beams undergo a mutual focusing or lensing effect due to energy exchange. If this occurs over a distance comparable to the diffraction length, i.e. $L_{\mathrm{D}} \geq L_{\mathrm{PG}}$, then the two effects can compensate resulting in a mutually locked soliton. This argument can be extended to the (detuned) phase-mismatched case, i.e. $(k_2 - 2k_1)L \neq 0$.