Steady-state pulse propagation in inhomogeneously broadened absorbers in the presence of a Kerr nonlinearity

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We consider the effect of a background dielectric on the steady-state propagation of a laser pulse through inhomogeneously broadened two-level atoms embedded in the dielectric. Because this problem apparently does not admit of analytic solutions, we have solved it numerically. A comparison of our results with the known analytic solutions for the homogeneously broadened atomic line shows that the two are similar. It seems that the new numerical solutions, unlike those for the homogeneous line, are not factorable, a fact to which we ascribe the difficulty in finding analytic solutions. We have found that as the atomic line broadens, the pulse widens and its height decreases, maintaining its area virtually constant.

INTRODUCTION

By Kerr nonlinearity we understand the effect that the background dielectric has on a pulse propagating through two-level atoms that are embedded in it. In the present study the nonlinearity of the dielectric manifests itself through the quadratic dependence of its index of refraction on the electric field of the pulse. Such a dependence has been considered in classical physics and is known to give rise to the Kerr effect.[1]

Eberly and Matulic[2] were the first to consider the effect of the Kerr nonlinearity on steady-state pulses in absorbers. They found that stable shape-preserving pulses of the type seen in self-induced transparency also exist in such media and that they must always exhibit phase modulation (or chirping). They considered only homogeneously broadened absorbers, because only in this case can the equations of motion describing the pulse–atom interaction be solved analytically. On the other hand, several attempts to solve analytically the equations for inhomogeneously broadened absorbers have led to contradictions.[2,3] In this paper we report some results of a numerical study of the propagation of a single pulse in inhomogeneous absorbers in the presence of a nonresonant nonlinearity of the Kerr type.

BACKGROUND

In the semiclassical approach to the problem of light–atom interactions we can use the slowly varying amplitude and phase approximation to write the equations governing this interaction in the following compact form:

$$\dot{u} = -(\Delta - \dot{\phi})v, \tag{1a}$$

$$\dot{v} = (\Delta - \dot{\phi})u + \Omega w, \tag{1b}$$

$$\dot{w} = -\Omega v, \tag{1c}$$

$$\dot{\Omega} = -{\nu_c}^2 \langle v \rangle, \tag{1d}$$

$$\Omega\dot{\phi} = {\nu_c}^2 \langle u \rangle + \tau_K \Omega^3. \tag{1e}$$

These equations describe the steady state of the pulse propagation, which is expected to be achieved after the pulse has penetrated several Beer's lengths into the medium. In that situation the atomic varibles u, v, and w and the field variables Ω and ϕ depend on space and time only through the local time variable $\zeta = t - z/V$, where V is the velocity of the pulse. In Eqs. (1) the overdots indicate differentiation with respect to ζ. We see from Eqs. (1a)–(1c) that the atomic variables must also depend on the parameter Δ, which represents the detuning of the individual atomic transition frequency ω relative to the carrier frequency of the laser beam ω_L, i.e., $\Delta = \omega - \omega_L$. The Rabi frequency Ω is defined as $2p\epsilon/\hbar$, where p is the atomic dipole moment and ϵ is the slowly varying envelope of the pulse, while ϕ represents its slowly varying phase. It is important to note that in Eqs. (1d) and (1e) the angle brackets represent the averaging of $u(\zeta, \Delta)$ and $v(\zeta, \Delta)$ over the inhomogeneous atomic line-shape function $g(\Delta)$, normalized in such a way that

$$\int_{-\infty}^{\infty} g(\Delta)\, d\Delta = 1. \tag{2}$$

Consequently the field variables Ω and ϕ are independent of the detuning Δ. The quantity ν_c has the dimensions of frequency, and it depends on the properties of the medium. Finally, the quantity τ_K, with the dimensions of time, represents the effects of the nonlinear interaction of the pulse with the background dielectric and is given by

$$\tau_K = \beta\omega_L\hbar^2/4c_0\delta p^2. \tag{3}$$

Equations (1) are valid only for small values of τ_K. In Eq.

(3) $\delta = 1/V - 1/c_0$ and $c_0 = c/\eta_0$ is the velocity of light in the dielectric whose nonlinear index of refraction is

$$\eta = \eta_0(1 + \beta\epsilon^2). \tag{4}$$

In all known materials the parameter β is a small quantity, of the order of 10^{-11} esu,[4] so that the term $\beta\epsilon^2$ is much smaller than 1, even for large fields. Equations (1) have been solved only for the homogeneously broadened case, i.e., where $g(\Delta) = \delta(\Delta)$.

In the absence of the term in τ_K, Eqs. (1) describe the bare self-induced transparency that has been studied extensively.[2] In particular, it was found that it is possible to obtain a large set of analytic solutions of this system of integrodifferential equations by assuming that the out-of-phase component of the atomic polarization, $v(\zeta, \Delta)$, is factorable, i.e., that it can be written as a product of a function of the local time ζ and a function of the detuning Δ:

$$v(\zeta, \Delta) = F(\Delta)v(\zeta, 0). \tag{5}$$

Admittedly the solutions obtained under this assumption constitute a special set of solutions of Eqs. (1). A prominent member of this set is the hyperbolic-secant function describing the single pulse, or soliton, of McCall and Hahn,[5] the only one that has yet been produced in the laboratory.[5,6] All other solutions represent shape-preserving pulse trains, and, to the best of our knowledge, they have not yet been experimentally observed. Their experimental realization will most likely require special preparation of the atomic medium, which may not be easily achieved.

The factorization assumption, Eq. (5), does not work in the presence of the Kerr nonlinearity; that is, it has not been found to be helpful in obtaining analytic solutions and actually leads to contradictions.[2,3] Several attempts to find analytic solutions to Eqs. (1) have ended in failure, so we found it appropriate to solve them numerically.

NUMERICAL RESULTS

We have solved numerically the problem of single-pulse propagation in inhomogeneous absorbers in the presence of a Kerr nonlinearity, assuming that all the resonant atoms are initially in the ground state. This is the most easily realizable situation in the laboratory and, in the absence of the Kerr term, leads to the well-known hyperbolic-secant solution of McCall and Hahn. The solution of Eqs. (1) for atoms in a nonresonant dielectric for which β is nonzero, for a sharp-line absorber, is essentially the same; the single major difference is that in the latter case the hyperbolic-secant pulse is always phase modulated (chirped), whereas in the former case this modulation is absent.

In our numerical studies, an important parameter is the standard deviation of the atomic line-shape function $g(\Delta)$, which we represent by σ. We have varied σ through a considerable range: from $\sigma = 0$ [$\ll \nu_c \approx (t_{1/2})^{-1}$], which, of course, corresponds to a homogeneously broadened atomic line, to $\sigma \approx \nu_c \approx 1$, which represents a fairly broad (inhomogeneous) atomic line. Here $t_{1/2}$ is the half-width at half-maximum of the pulse Ω.

When we set $\sigma = 0$, we obtained a solution that coincides with the analytic solution found by Eberly and Matulic; this corroborates our understanding that the system of Eqs. (1) has a unique solution for a given set of initial conditions.[7] Figure 1 is an example of this case. It exhibits the computer solution of Eqs. (1) for the envelope Ω, the chirp $\dot\phi$, and the inversion w for a sharp-line case ($\sigma = 0$), with Kerr constant $\tau_K = 0.01$ and $\nu_c = 1.5$; these results are distinguishable from the ones obtained analytically. For simplicity we suppress

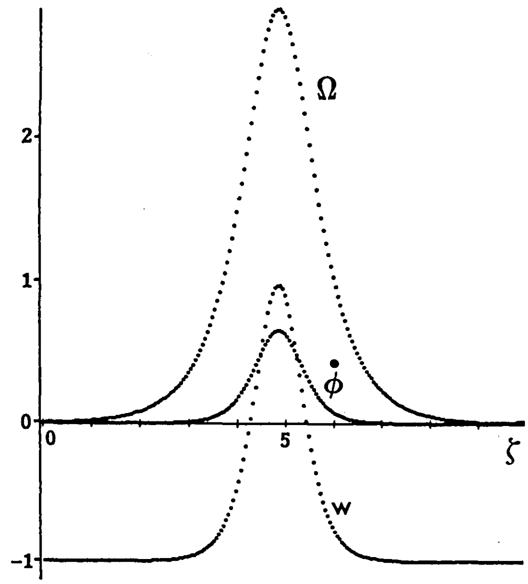

Fig. 1. The dimensionless envelope Ω, chirp ϕ, and inversion w for a sharp-line ($\sigma = 0$) case with $\nu_c = 1.5$ and Kerr constant $\tau_K = 0.01$. In the limit $\tau_K \to 0$, the chirp vanishes everywhere, but Ω and w are not changed appreciably. The chirp $\dot\phi$ is magnified 25 times.

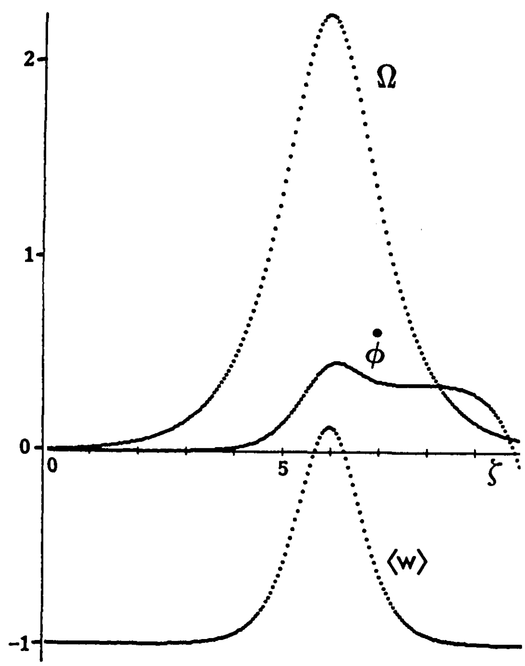

Fig. 2. The dimensionless envelope Ω, chirp ϕ, and average inversion $\langle w\rangle$ for a broad-line case: $\sigma = \nu_c = 1.5$. The value of the Kerr constant is $\tau_K = 0.01$ (as in Fig. 1). The atomic line shape $g(\Delta)$ is Gaussian, with standard deviation σ. The chirp $\dot\phi$ is magnified 25 times.

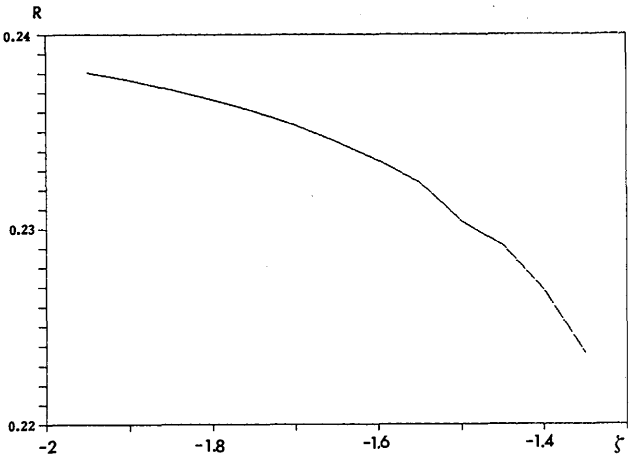

Fig. 3. The ratio $R \equiv v(\zeta, \Delta)/v(\zeta, 0)$ as a function of the local time ζ for $\Delta = -2\sigma$; $\tau_K = 0.01$, and $\sigma = 1.5$. On the same graph, a numerical solution for which $\tau_K = 0$ would yield a ratio R that would not differ visibly from a straight horizontal line.

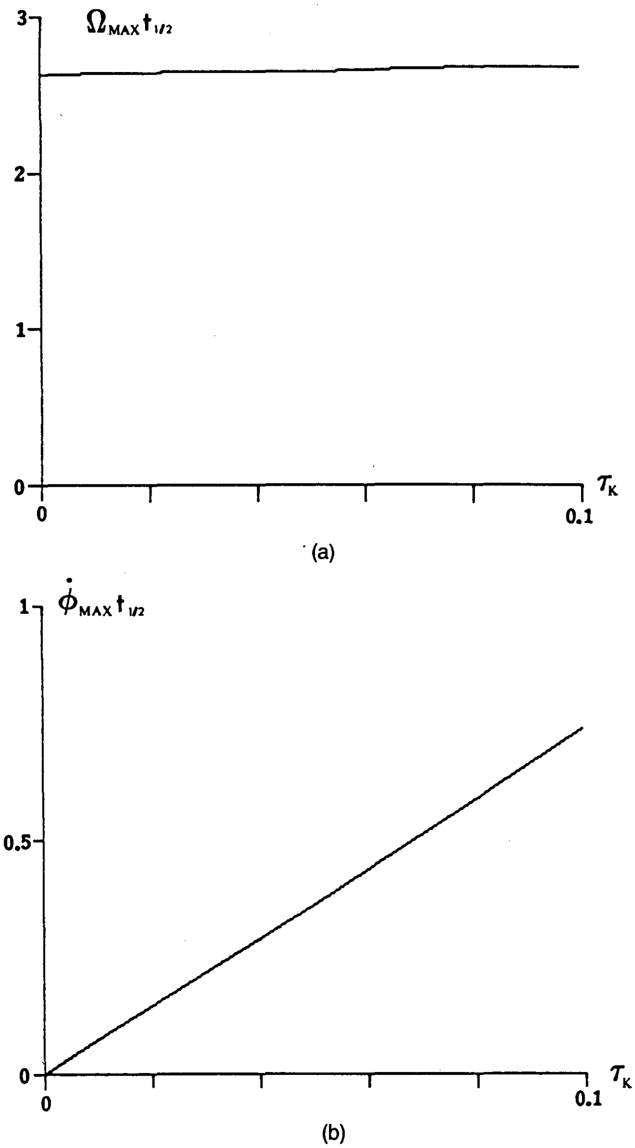

Fig. 4. The products (a) $\Omega_{\text{MAX}}t_{1/2}$ and (b) $\dot{\phi}_{\text{MAX}}t_{1/2}$ as functions of τ_K; $\nu_c = \sigma = 1.5$.

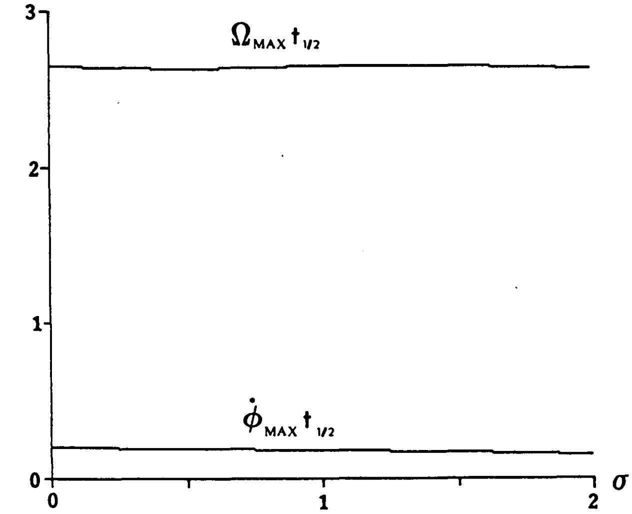

Fig. 5. The products $\Omega_{\text{MAX}}t_{1/2}$ and $\dot{\phi}_{\text{MAX}}t_{1/2}$ as functions of the standard deviation of the atomic line shape σ; $\tau_K = 0.01$, $\nu_c = 1.5$.

the units of all variables: time units may be introduced to τ (and hence τ_K and ζ), from which the frequency units at Ω, ν_c, σ, Δ, and $\dot{\phi}$ are found.

Figure 2 contains the plots of Ω, $\dot{\phi}$, and $\langle w \rangle$ for the broad-line case for which $\sigma = \nu_c = 1.5$ and the same value of τ_K is used as in Fig. 1. [The atomic line $g(\Delta)$ has a Gaussian shape in all examples.] The only major difference between these curves and those for which $\tau_K = 0$ is the presence of a small asymmetric chirp when the Kerr dielectric is present. We conclude, therefore, that shape-preserving pulses may propagate through two-level absorbers even in the presence of nonresonant background dielectrics. An important difference between Kerr pulses and those of bare self-induced transparency, however, is that the former are not factorable. We see this clearly in Fig. 3, in which we show the ratio $R \equiv v(\zeta, \Delta)/v(\zeta, 0)$ along a portion of the ζ axis, for the case $\tau_K = 0.01$ and $\sigma = 1.5$. In the case $\tau_K = 0$ the ratio R is constant with respect to ζ, indicating that the factorization [Eq. (5)] holds; in the Kerr case this ratio is not strictly constant in ζ. The presence of a small term in τ_K destroys the factorability of the absorptive component of the atomic polarization without preventing the formation of the pulse. We conclude that the factorization is not merely a mathematical ansatz but that it describes a particular physical situation, as evidenced by the pulses and pulse trains of bare self-induced transparency. In light of the analytical inconsistencies to which the factorization assumption leads, we understand how the numerical Kerr solutions do not display factorability: such solutions simply do not exist. We have seen, however, that Eqs. (1) do admit of nonfactorable solutions, which represent propagation of shape-preserving pulses in more complicated situations. We think that the original experiments of McCall and Hahn, with the pulses propagating in a ruby rod, may be examples of pulses described by the present theory.

In Figs. 4(a) and 4(b) we have plotted $\Omega_{\text{MAX}}t_{1/2}$ and $\dot{\phi}_{\text{MAX}}$ $t_{1/2}$ against τ_K. Figure 4(a) indicates that $\Omega_{\text{MAX}}t_{1/2}$ is virtually independent of τ_K (for small τ_K). Figure 4(b) shows that $\dot{\phi}_{\text{MAX}}t_{1/2}$ depends linearly on τ_K, a result that could have been expected from the analogous analytical behavior.[2]

Finally, in Fig. 5, we exhibit the dependence of $\Omega_{\text{MAX}}t_{1/2}$

and $\dot{\phi}_{MAX}t_{1/2}$ on σ (the width of the atomic line). The constancy of these quantities reflects the fact that the pulse Ω and the chirp $\dot{\phi}$ broaden (and their maxima decrease) as the atomic line widens, maintaining a constant pulse area.

CONCLUSIONS

Summarizing, we can say that our numerical studies of the optical Maxwell–Bloch equations in which a Kerr term has been added show that the propagation of undistorted pulses through resonant inhomogeneously broadened absorbers in the presence of a nonlinear background dielectric is theoretically possible. These pulses exhibit similar properties to those propagating through media in which the dielectric is absent. Solutions for the absorptive component of the atomic polarization, however, are not strictly factorable, a fact that explains why such an assumption failed to produce self-consistent analytic solutions. The original experiments of McCall and Hahn, in which light pulses propagated through chromium ions embedded in a background dielectric of aluminum oxide, are examples of pulses described in this paper.

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ANNOUNCEMENTS

Second International Conference on the Electrical Transport and Optical Properties of Inhomogeneous Media—ETOPIM2

Paris, France, August 29–September 2, 1988

This conference is a follow-up of the 1977 Columbus conference on the same topic. Although the intervening years have seen conferences with closely related themes, there has been no direct successor to the first ETOPIM conference. Moreover, the field of inhomogeneous media has broadened significantly during the past ten years, with new topics such as electron and photon localization, new approaches to coherence, nonlinear effects, percolation, optical cross over, thermal transport, relation to morphology, fractal and multifractal concepts, and many new materials. This conference will bring together persons working on different approaches: fundamental (theory and experiment) and applications. Invited papers will focus on the new subjects recently developed and will review some important, broader themes. Contributed papers on these new topics, covering original unpublished work, are particularly encouraged. For more information, write to the ETOMIM2 Secretary, Jacques LAFAIT, Laboratoire d'Optique des Solides, Université Pierre and Marie Curie, 4, Place Jussieu, 75252 Paris Cedex 05, France. Phone, (1) 43 36 25 25 4353 or 3981; Telex, UPMCSIX 200145 F; Teletex, 933-146333527-UPMCSIX; Bitnet, LAFAIT AT FRCPN11.

Euroanalysis VII

Vienna, Austria, August 26–31, 1990

Euroanalysis VII will emphasize methodological developments, especially those in analytical chemistry, for problem solving in major areas of science: clinical, pharmaceutical, forensic, food, agricultural; in environmental chemistry, materials science, biotechnology, arts, and archeology. Topical issues such as computer-based analytical chemistry will also be addressed. A large instrument exhibit is planned. For information and registration, write to M. Grasserbauer, c/o Interconvention, Austria Center Vienna, A-1450 Vienna, Austria. Phone: +43-222-2369/647; Telex: 11 18 03 ICOS A; cable: INTECON WIEN; FAX: +43-222-2369/648.

12th International Congress of X-Ray Optics and Microanalysis

Cracow, Poland, August 28–September 1, 1989

The 12th International Congress of X-Ray Optics and Microanalysis will be held on August 28–September 1, 1989, in Cracow. For information, write to Stanislawa Jasienska, Institute of Metallurgy, Academy of Mining and Metallurgy, al Mickiewicza 30, 30-59 Cracow, Poland. Phone: 338100, ext. 2917; Telex 0322202AGHPL.

TECHNICAL CALENDAR

Meetings of the Optical Society of America*

1988

August 22–25 **OSA Topical Meeting on Nonlinear Optical Properties of Materials,** Troy, New York.

August 30–September 2 **ICO International Topical Meeting on Optical Computing,** Toulon, France. Information: G. Roblin, Technical Program Committee Secretary, Institut d'Optique, BP 43, 91406 Orsay Cedex, France.

September 6–10 † **Annual Meeting of the International Neural Network Society,** Boston, Massachusetts. Information: INNS Abstracts 1988, 10615G Tierrasanta Blvd., Suite 346, San Diego, California 92124.

September 11–15 † **14th European Conference on Optical Communication,** Brighton, United Kingdom. Information: ECOC '88 Secretariat, Conference Services, IEE, Savoy Place, London WC2R OBL, United Kingdom.

September 26–29 **OSA Topical Meeting on Short-Wavelength Coherent Radiation,** North Falmouth, Cape Cod, Massachusetts.

September 27–29 **OSA Topical Meeting on Space Optics for Astrophysics and Earth and Planetary Remote Sensing,** North Falmouth, Massachusetts.

October 31–November 4 **Optics '88: OSA Annual Meeting,** Santa Clara, California.

November 2–4 **OSA Workshop on Optical Fabrication and Testing,** Santa Clara, California.

December 19–21 † **Sixth Meeting on Optical Engineering in Israel,** Tel Aviv, Israel.

1989

January 16–18 **LEOS-IEEE/OSA/SPIE Topical Meeting on Optical Data Storage,** Los Angeles, California.

February 2–4 **LEOS-IEEE/OSA Topical Meeting on Nonlinear Guided-Wave Phenomena: Physics and Applications,** Houston, Texas.

February 6–9 **LEOS-IEEE/OSA/SPIE Topical Meeting on Integrated and Guided-Wave Optics (IGWO '89),** Houston, Texas.

February 6–9 **LEOS–IEEE/OSA Conference on Optical Fiber Communication (OFC '89),** Houston, Texas.

February 13–16 **OSA Topical Meeting on Noninvasive Assessment of the Visual System,** Santa Fe, New Mexico.

February 15–17 **OSA Topical Meeting on High-Resolution Optical Fourier-Transform Spectrometry,** Santa Fe, New Mexico.

March 1–3 **LEOS–IEEE/OSA Topical Meeting on Photonic Switching II,** Salt Lake City, Utah.

March 6–8 **LEOS–IEEE/OSA Topical Meeting on Quantum Wells for Optics and Optoelectronics,** Salt Lake City, Utah.

April 24–28 **LEOS–IEEE/OSA Conference on Lasers and Electro-Optics (CLEO '89),** Baltimore, Maryland.

July 12–14 † **Topical Meeting on Solid-State Lasers,** Beijing, China. Information: Y. Peide, Beijing University of Posts and Telecommunications, Beijing, China. (Abstract deadline: December 1, 1988.)

July 18–21 † **Seventh International Conference on Integrated Optics and Optical Fiber Communication (IOOC '89),** Kobe, Japan. Information: Secretariat, IOOC '89, c/o Business Center for Academic Societies Japan, Conference Department, Yamazaki Bldg. 4F, 2-40-14, Hongo, Bunkyo-ku, Tokyo 113, Japan.

July 24–26 **Second Microoptics Conference/Eighth Topical Meeting on Gradient-Index Optical Imaging (MOC/GRIN),** Tokyo, Japan. Information: Kenichi Iga, General Chair, Tokyo Institute of Technology, Research Laboratory of Precision Machinery and Electronics, 4259 Nagatsuta, Midori-ku, Yokohama 227, Japan.

September 11–15 **European Conference on Optical Communication (ECOC '89),** Göteborg, Sweden. Information: ECOC '89 Secretariat, Department of Electrical Measurements Research Laboratory of Electro-Optics and Laser, Chalmers Institute of Technology, S-41296 Göteborg, Sweden. (Abstract deadline: March 8, 1989.)

October 15–20 **Optics '89: OSA Annual Meeting,** Orlando, Florida.

1990

January 12–26 **LEOS-IEEE/OSA Conference on Optical Fiber Communication (OFC '90),** San Francisco, California.

May 21–25 **LEOS-IEEE/OSA Conference on Lasers and Electro-Optics (CLEO '90),** Anaheim, California.

* OSA, Optical Society of America; ICO, International Commission for Optics; LEOS, Lasers and Electro-Optics Society; IEEE, Institute of Electrical and Electronics Engineers; SPIE, Society of Photo-Optical Instrumentation Engineers.
† OSA is a cooperating society.

FEATURE CALENDAR—JOSA A

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