Basic emergence of dispersive and nonlinear effects in fibers for supercontinuum generation by ultrashort pulses

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Abstract: We demonstrate analytically and numerically that highly nonlinear optical fibers (HNLFs) shows evidence of new kind of dispersive and nonlinear phenomena when a basic approximation $\beta(\omega)+\beta(\omega_0)\approx 2\beta(\omega_0)$ [1] is not taken into account for the supercontinuum (SC) generation model.

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1. Introduction

In this work we have analytically and numerically investigated the consequences of do not consider the basic approximation $\beta(\omega)+\beta(\omega_0)\approx 2\beta(\omega_0)$ as a rule used for modeling the generalized nonlinear Schrodinger equation (GNLSE) [1]. In this approach β is the propagation constant that corresponds to the single-mode fiber, and ω_0 is the carrier frequency. This is a partial intuitive consideration for the propagation of ultrashort optical pulses through HNLFs. For this instance, we have modeled the SC generation considering vectorial mode amplitude equations within the frame of the slowly varying envelope approximation. The processes, theoretically studied, suggest a new apparent factor in the classical GNLSE [1,2]. In this way, we observe, analytically and numerically, more general dispersive and nonlinear effects in fibers for SC generation by ultrashort pulses. We report the modified GNLSE and the numerical results of a particular behavior of the SC generation with femtosecond pulses.

2. Derivation of the modified GNLSE

The starting point for the modified GNLSE derivation is the Maxwell frequency-domain wave equation for the optical field in a silica like optical medium, valid for both linear and nonlinear characterization, which is given by

$$-\nabla(\nabla\cdot\tilde{\mathbf{E}})+\nabla^2\tilde{\mathbf{E}}+k^2n^2(x,y)\tilde{\mathbf{E}}=-\mu\omega^2\tilde{\mathbf{P}}_{NL}, \tag{1}$$

where we have included the term $\nabla(\nabla\cdot\tilde{\mathbf{E}})$ because of its significance in HNLFs [1]. Here k is the free-space wavenumber, n is the refractive index profile of the waveguide, and $\tilde{\mathbf{P}}_{NL}$ is the nonlinear polarization. When an optical pulse propagates inside a single-mode fiber, its electric field $\tilde{\mathbf{E}}$ can be expressed by [1]

$$\tilde{\mathbf{E}}(\mathbf{r},\omega)=\frac{1}{2}\left\{\mathbf{F}(x,y,\omega)\tilde{a}(z,\omega)\exp(i\beta_0 z)+\text{c.c.}\right\}, \tag{2}$$

where $\mathbf{F}=\mathbf{e}/\sqrt{N}$ and N is related to the spectral power obtained using the Poynting vector [3]. Here $\mathbf{e}=\mathbf{e}(x,y,\omega)$ is the mode structure of the fiber, $\tilde{a}$ is the slowly varying modal amplitude in the frequency domain, and β_0 is the propagation constant of the field. The mode solution satisfies the equation: $-\nabla[\nabla\cdot\mathbf{e}e^{i\beta z}]+[\nabla^2+k^2n^2]\mathbf{e}e^{i\beta z}=0.$ Substituting Eq. (2) into Eq. (1), applying the slowly varying envelope approximation and assuming that the light is spectrally broad, we obtain after associating and including terms

$$\frac{\partial a}{\partial z}=i\frac{1}{2}(1+\delta)\left(1+i\tau_{shock}\frac{\partial}{\partial t}\right)\left(\sum_{n=1}^{\infty}i^n\frac{\beta_n}{n!}\frac{\partial^n}{\partial t^n}\right)a+\delta\left(1+i\tau_{shock}\frac{\partial}{\partial t}\right)^2\frac{i\omega_0 e^{-i\beta_0 z}}{2\sqrt{N}}\iint\mathbf{e}^*\cdot\mathbf{P}_{\mathrm{NL}}\,dxdy \tag{3}$$

where $\delta = N\omega_0\mu_0 \big/ [\beta_0 \iint |\mathbf{e}_T|^2 dxdy - i\iint (\mathbf{e}_T \cdot \nabla_T e_z^*) dxdy]$, $\tau_{shock} = 1/\omega_0$, and β_n are the dispersion coefficients defined as $\beta_n = (\partial^n \beta / \partial \omega^n)\big|_{\omega=\omega_0}$. When we treat with HNLFs which its core size is not comparable to or shorter than the optical wavelength, $\delta \approx 1$. Following an analog direction like in Ref [1] to derive the GNLSE, we obtain

$$\frac{\partial a}{\partial z} + \frac{\alpha}{2}A + i\frac{\beta_2}{2}\frac{\partial^2 a}{\partial T^2} - \sum_{n\geq 2}\frac{i^{n+2}}{n!}\left\{\frac{\beta_{n+1}}{(n+1)} + \tau_{shock}\beta_n\right\}\frac{\partial^{n+1} a}{\partial T^{n+1}} = i\gamma\left(1 + i2\tau_{shock}\frac{\partial}{\partial T} - \tau_{shock}^2\frac{\partial^2}{\partial T^2}\right) \times\left(a(z,T)\int_{-\infty}^{\infty} R(T')\left|a(z,T-T')\right|^2 dT'\right) \tag{4}$$

which is the modified GNLSE that describes the behavior of light inside HNLFs. Here we have assumed that the mode profile $\mathbf{e}(x, y, \omega)$ do not chance significantly over the pulse bandwidth from its value at the carrier frequency ω_0.

3. Results

We show particular numerical results with the idea of evidence the basic modified behavior of the SC generation phenomenon in HNLFs. We show a comparison between simulations done with the classical GNLSE [1,2] and with the modified GNLSE (Eq. (4)). Figures 1 and 2 show the differences in propagation and output occurring in the SC generation simulation process. For simplicity, we have used the fiber and input pulse parameters of Ref. [2].

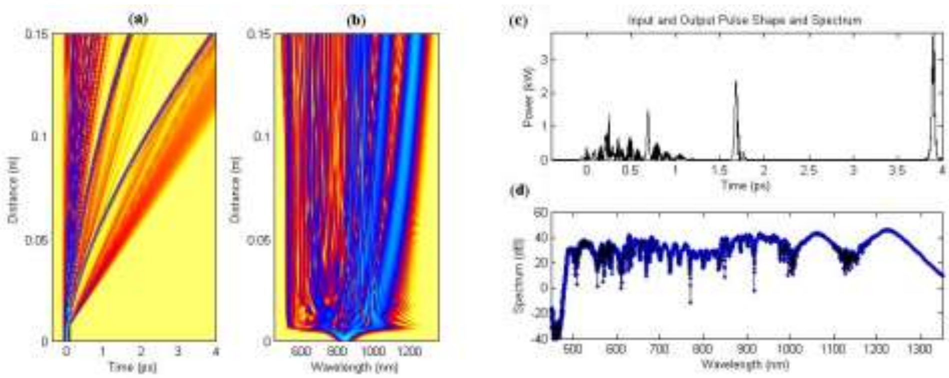

Fig. 1. (a) Temporal and (b) spectral evolutions of a pulse launched with $N \approx 8.6$ over a distance of $L = 15cm$. (c) Output temporal intensity and (d) spectra. The input pulse peak power is 10 *kW*, $\tau_{shock} = 0.56$ *fs*, and duration (FWHM) is 50 *fs* [2].

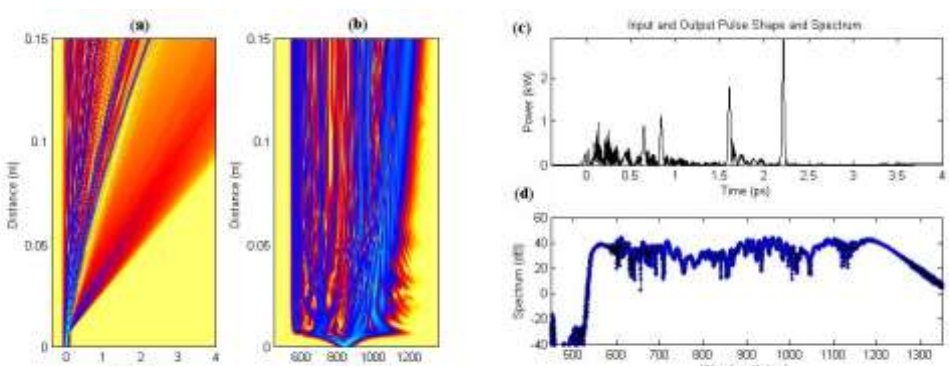

Fig. 2. (a) Modified temporal and (b) spectral evolutions of a pulse launched with $N \approx 8.6$ over a distance of $L = 15cm$. (c) Modified output temporal intensity and (d) spectra. The input pulse peak power is 10 *kW*, $\tau_{shock} = 0.56$ *fs*, and duration (FWHM) is 50 *fs* [2].

4. Conclusions

In this work we have demonstrated analytically and numerically that HNLFs shows evidence of new kind of dispersive and nonlinear phenomena when a basic approximation $\beta(\omega)+\beta(\omega_0)\approx 2\beta(\omega_0)$ is not taken into account for the supercontinuum (SC) generation model. We have modeled the SC generation considering a new apparent factor in the classical GNLSE. We have observed, analytically and numerically, more general dispersive and nonlinear effects in fibers for SC generation by ultrashort pulses. Also, we have reported the modified GNLSE, and the numerical results of a particular behavior of the SC generation with femtosecond pulses.

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