Entangled revivals in a Cross Cavity Jaynes-Cummings Model.

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Abstract: We analyze the generation of photonic entanglement in a two-level atom placed in a cross cavity, by means of the Jaynes Cummings model. Our configuration proved to be a suitable tool to study entangled revivals.

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1. Introduction

In this work, we describe a variation of the Jaynes-Cummings model [1] consisting of a two-level atom (TLA) placed in a cross cavity with symmetrical arms. Each arm of the cavity has its own and independent modes, nevertheless, they can have the form of path-entangled states.

2. Mathematical model and solution

The Hamiltonian of the described system is given by

$$H=\frac{1}{2}\hbar\omega_0\sigma_z+\hbar\omega\left(a^\dagger a+b^\dagger b\right)+\hbar\left[g_x\left(a+a^\dagger\right)+g_y\left(b+b^\dagger\right)\right]\sigma_+ +\hbar\left[g_x\left(a+a^\dagger\right)+g_y\left(b+b^\dagger\right)\right]\sigma_- . \tag{1}$$

The frequency of the atom is denoted by ω_0 and the frequency of the fields is ω. The annihilation (creation) operators of each arm are given by $a(a^\dagger)$ and by $b(b^\dagger)$ while the coupling of the atom with the cavity is given by g_x and g_y in the x and y directions respectively. A pair of new operators, such that $\left[A,A^\dagger\right]$, $\left[B,B^\dagger\right]=1$ and $\left[A,B\right]=0$, are given by the equation (2):

$$\begin{aligned}A&=\cos\left(\frac{\theta}{2}\right)a+\sin\left(\frac{\theta}{2}\right)b,\\ B&=-\sin\left(\frac{\theta}{2}\right)a+\cos\left(\frac{\theta}{2}\right)b.\end{aligned} \tag{2}$$

That allows us to the RWA Hamiltonian in terms of the operators given in equation (2):

$$H_T=\hbar\bar{\omega}B^\dagger B+\hbar\bar{\omega}\left(\sigma_+\sigma_-+A^\dagger A\right)+\hbar\left[\frac{1}{2}\Delta\sigma_z+g_{\text{eff}}\left(A\sigma_++A^\dagger\sigma_-\right)\right]=\hbar\omega B^\dagger B+\hbar\omega N+\hbar C, \tag{3}$$

Where the effective coupling constant is given by $g_{\text{eff}}=\sqrt{g_x^2+g_y^2}$. The operators N and C defined by Ackerhalt (et al) [2], commute with the Hamiltonian $\left[H_T,C\right]=\left[H_T,N\right]=0$. The Hamiltonian has the structure of the well-known Jaynes-Cummings model, whose time evolution can be determined via the evolution operator $U_T(t)=e^{-i\omega B^\dagger Bt}e^{-i\omega Nt}e^{-iCt}$, and the field states can be rewritten using equation (2). In particular for a Fock state in the first arm, and vacuum in the other:

$$\left|m,0\right\rangle=\sum_{k=0}^{m}\frac{\sqrt{m!}}{\sqrt{(m-k)!}\sqrt{k!}}\left(\cos\frac{\theta}{2}\right)^{m-k}\left(-\sin\frac{\theta}{2}\right)^{k}\left|e,m-k,k\right\rangle\rangle . \tag{4}$$

We have demonstrated that this state shows revivals and it has become an ideal tool to analyze entanglement in the simplest cases i.e. an initial photonic state in the form of a NOON state, $\frac{1}{\sqrt{2}}\left(\left|m,0\right\rangle+\left|0,m\right\rangle\right)$.

3. Results

In order to understand the revivals dynamics of a cross cavity JCM, we need to know the time evolution of the atomic inversion and also that of the photon number in each arm, n_x and n_y. If the atom is initially excited and the field is prepared in the state given by equation (4) we find the corresponding expression for the atomic inversion:

$$w(t)=\sum_{k=0}^{m}\frac{m!}{(m-k)!k!}\left(\cos^2\frac{\theta}{2}\right)^{m-k}\left(\sin^2\frac{\theta}{2}\right)^{k}\left\{1-\sin^2\left(\frac{\Omega_{mk}}{2}t\right)\left[1+\left(\frac{\Delta}{\Omega_{mk}}\right)^2-\left(\frac{2g_{\text{eff}}}{\Omega_{mk}}\right)^2(m-k+1)\right]\right\}. \tag{5}$$

That shows revivals in the large photon limit [3], Fig (1a).

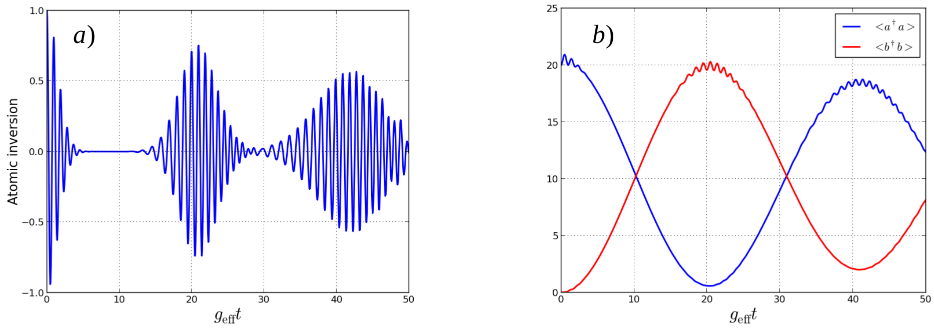

Figure 1. a) Revivals in the atomic inversion for the initial state given in equation (4) and the atom initially excited. b) Expected value of the number of photons $\langle a^\dagger a\rangle$ (red line) and $\langle b^\dagger b\rangle$. In both cases $m=20$.

It is quite interesting to notice the behavior of the average photon number in each arm, n_x and n_y, that reminds ourselves of a mode coupled system and therein the distinctive revival distribution, when the arm cavity field is filled, behavior hidden in the atomic inversion revival sequence and quite different from that of an initial entangled system, Fig, 2.

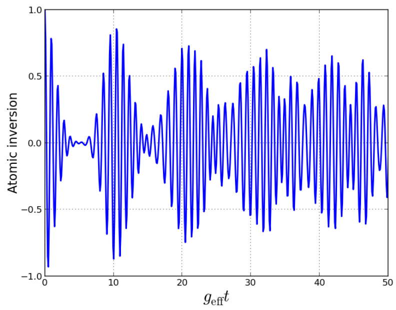

Figure 2. Atomic inversion of a field state $\frac{1}{\sqrt{2}}\left(|m,0\rangle+|0,m\rangle\right)$ with $m=20$. In this case the time of revival decreases

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