



## Complex dispersion relation of 1D dielectric photonic crystal with thin metallic layers

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### ABSTRACT

We have carried out the analytical and numerical analysis of metallic insets embedded in a dielectric photonic crystal (DPC). The corresponding one-dimensional metallo-dielectric photonic crystal (MDPC) is studied in relation to the substratum DPC. We describe the complex MDPC dispersion relation curves and the transmission coefficient at the different propagation regimens through the finite stack. While in the DPC, the band gap argument corresponds to undefined complex solutions of a real dispersion relation, in a MDPC corresponds to a definite complex solution, function of the metal thickness; and analytic continuation of the dispersion curve on the imaginary  $K$  plane. The metal absorption is described by the Drude model, and as a result of the absorption in the inset metallic films, complex Bloch vectors are produced for all frequencies as evanescent waves that differ from the straightforward metal absorption.

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### 1. Introduction

Nowadays, the dielectric photonic crystals (DPC) have been widely studied and received a great deal of attention because of its perceived applications, properties and new physical phenomena. In addition, the introduction of periodically spaced thin metallic elements is becoming quite attractive at nanodimensions, where metamaterials have already shown quite interesting phenomena.

Metallo-dielectric photonic crystals (MDPC), as basic stacks of a single dielectric with metallic insets at very low filling fractions (less than 1%), were first studied by Kuzmiak and Maradudin [1] using dispersion relations as approximate solutions. Yablonovitch and co-workers [2] studied them by the transfer matrix at metallic thickness that exceeded 10 nm, well beyond such early approach, demonstrating features that were clearly non perturbative at the dielectric substrate. All those single dielectric structures [1–5] have no paragon with basic DPCs, were both dispersion relation and  $M$  matrix methods are equivalent. We discuss in this work, the complementary of the dispersion relation and the transfer matrix method transmittance [6] of a particular one-dimensional (1D) MDPC, produced by a DPC with extremely thin metallic insets with the same periodicity. Nowadays, both types of basic structures (DPC and MDPC of metallic insets in a

dielectric) are well known as well as their basic features, and we expect them to coexist if the metallic insets are indeed extremely thin ( $< 10$  nm for Ag).

### 2. Analysis of the 1-D metallo-dielectric photonic crystal

The structure of the MDPC is two layers of different dielectric materials, with refractive index  $n_1$  and  $n_2$ , and metallic insets described by the Drude model:

$$\epsilon_3(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)} \quad (1)$$

as usual [1,2],  $\omega_p = 1.6 \times 10^{16} \text{ s}^{-1}$ , is the plasma frequency and  $\gamma = 0.001 \omega_p$  is the damping coefficient. The thickness of each layer is  $a$ ,  $b$  and  $\delta$ , and the cell period is  $L = a+b+\delta$ . The transmittance of the system, computed using the transfer matrix method [7], is given by:

$$T_n = \frac{1}{|M_{22}|^2} \quad (2)$$

the  $M_{22}$  element of the transfer matrix, from the first layer to the  $n$ th layer. DPC used as substrate has the dispersion relation:

$$\cos(\kappa L) = f(k_1, k_2) = \cos(k_1 a) \cos(k_2 b) - \frac{k_1^2 + k_2^2}{2k_1 k_2} \sin(k_1 a) \sin(k_2 b), \quad (3)$$

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where  $\kappa$  is the Bloch wave vector and  $k_1$  and  $k_2$  are the wave vectors in the respective medium. On the other hand, in the chosen MDPC the dispersion relation is given in two no-redundant equations [1]; the real part given by:

$$\begin{aligned} &\cos(\kappa_R L) \cosh(\kappa_I L) \\ &= f(k_1, k_2) \cos(k_{3R} \delta) \cosh(k_{3I} \delta) - \frac{1}{2|k_3|^2} \\ &\quad \times \alpha[k_{3R} \sin(k_{3R} \delta) \cosh(k_{3I} \delta) + k_{3I} \cos(k_{3R} \delta) \sinh(k_{3I} \delta)] \\ &\quad - \frac{1}{2k_1 k_2} \beta[k_{3R} \sin(k_{3R} \delta) \cosh(k_{3I} \delta) \\ &\quad - k_{3I} \cos(k_{3R} \delta) \sinh(k_{3I} \delta)], \end{aligned} \tag{4}$$

whereas the imaginary part is given by:

$$\begin{aligned} &\sin(\kappa_R L) \sinh(\kappa_I L) \\ &= f(k_1, k_2) \sin(k_{3R} \delta) \sinh(k_{3I} \delta) + \frac{1}{2|k_3|^2} \\ &\quad \times \alpha[k_{3R} \cos(k_{3R} \delta) \sinh(k_{3I} \delta) - k_{3I} \sin(k_{3R} \delta) \cosh(k_{3I} \delta)] \\ &\quad + \frac{1}{2k_1 k_2} \beta[k_{3R} \cos(k_{3R} \delta) \sinh(k_{3I} \delta) \\ &\quad + k_{3I} \sin(k_{3R} \delta) \cosh(k_{3I} \delta)]. \end{aligned} \tag{5}$$

In Eqs. (4) and (5), the function  $f(k_1, k_2)$  is the original dispersion relation of the DPC substrate, Eq. (3). Other terms are introduced

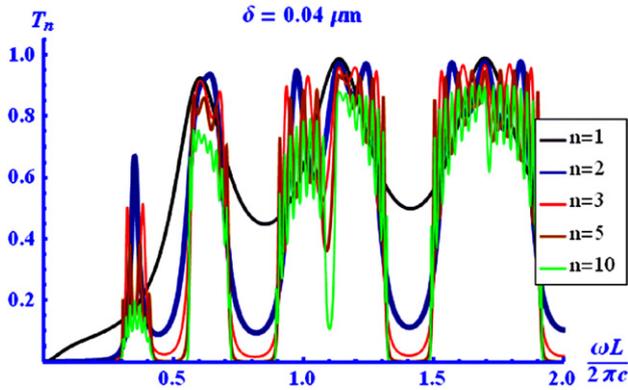


Fig. 1. The periodic metal array changes the concept of skin depth and partially suppress the absorption as the field propagates within the material.

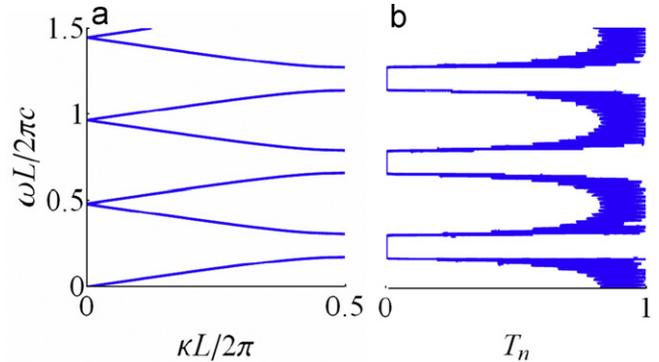


Fig. 3. For the DPC substrate  $\delta = 0$ : (a) the band structure; (b) the stopgap for 10 periods. In both pictures  $a = 0.1082 \mu\text{m}$ ,  $b = 0.2654 \mu\text{m}$  and  $n_1 = 3.58$  and  $n_2 = 1.46$ .

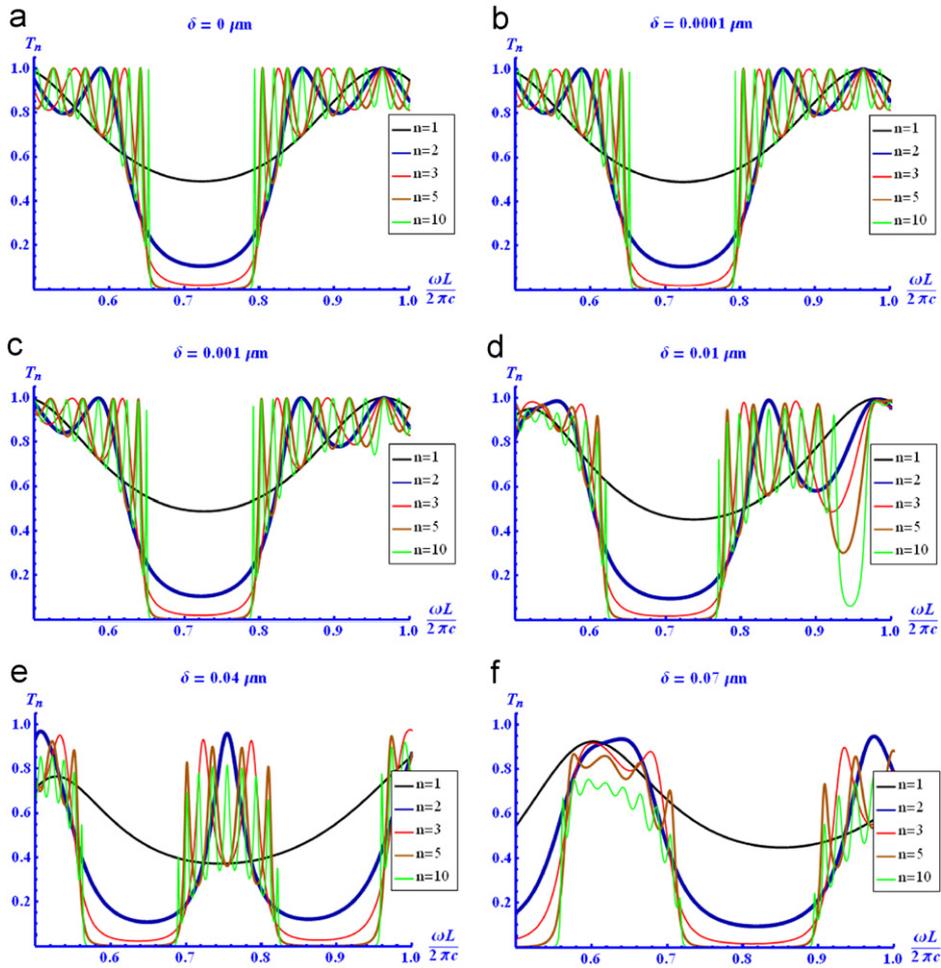


Fig. 2. The stopgap is completely formed with only 10 primitive cells,  $n_1 = 3.58$ ,  $n_2 = 1.46$  and  $n_3$  follows the Drude model:  $a = 0.1082 \mu\text{m}$ ,  $b = 0.2654 \mu\text{m}$  and (a)  $\delta = 0$ , (b)  $\delta = 0.0001 \mu\text{m}$ , (c)  $\delta = 0.001 \mu\text{m}$ , (d)  $\delta = 0.01 \mu\text{m}$ , (e)  $\delta = 0.04 \mu\text{m}$  and (f)  $\delta = 0.07 \mu\text{m}$ .

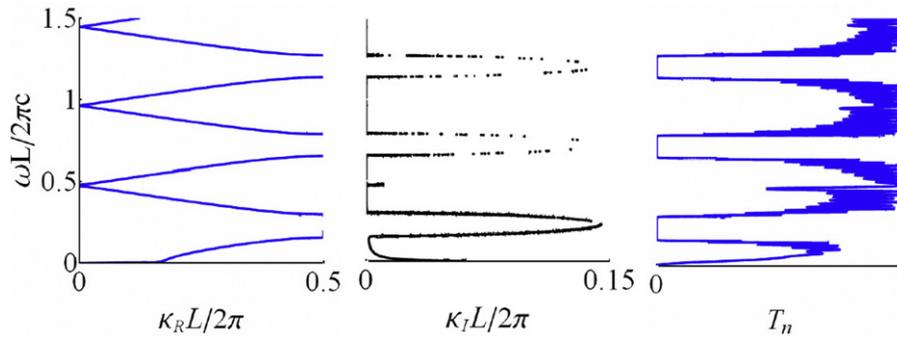


Fig. 4. The stopgap, band structure and the absorption for a MDPC for the same parameters that in Fig. 3, in this case  $\delta = 0.001 \mu\text{m}$ .

by the metallic layer and  $\alpha$  and  $\beta$  are given by:

$$\alpha = k_1 \sin(k_1 a) \cos(k_2 b) + k_2 \cos(k_1 a) \sin(k_2 b) \quad (6)$$

and

$$\beta = k_2 \sin(k_1 a) \cos(k_2 b) + k_1 \cos(k_1 a) \sin(k_2 b). \quad (7)$$

In this case, we have a complex wave vector for the metal with real (imaginary) part  $k_{3R}$  ( $k_{3I}$ ) and the same happen for the Bloch wave vector  $\kappa_R$  ( $\kappa_I$ ). Simultaneously, solving Eqs. (4) and (5), we find the dispersion relation for the real part of the wave number,  $\kappa_R$  (refractive index), as well as the dispersion relation for the imaginary part  $\kappa_I$  (the absorption).

For the numerical simulations, we have chosen the central wavelength as  $\lambda_0 = 1.55 \mu\text{m}$ , a telecommunication frequency, the thickness of each dielectric layer as  $\lambda/4$  and  $\delta$ , the thickness of the metallic layer, as the parameter of both the transmission profile and the dispersion relations. In Fig. 1, we describe the inhibition of the stack absorption, due to the transmission–reflection feedback at each additional metallic layer, as the field propagates through the material [3]. With a single or just a few cells, there is attenuation at low frequencies as expected for a metal, but after two cells we can notice that there is an enhancement of the expected transmission. In Fig. 2, we can notice the shift in the central point of the stopgap as well as its broadening, in this case as a function of the number of primitive cells  $n$ , not just as a function of the metal thickness [6].

Due to the high contrast between the two dielectrics, with only 10 primitive cells there is an excellent convergence in the transmittance and the stopgap is already formed. In practice, this and the excellent match between the transmittance and the formed band structure, allows us to consider them as already corresponding to an ideal infinite crystal. In the limit of very thin metal layers, we may compare both the band and transmission diagrams for different metal thickness  $\delta$ . Then, we realize that in the distinction from very thin to thin metallic insets resides the difference between Refs. [1] and [2], where our DPC substrate has demonstrated the interaction between the dielectric and metallic behaviors and pointed out, that the range of 10 nm metal

thickness is the boundary for both behaviors. This is shown in Figs. 3 and 4. The band gaps in extremely thin metal have been explored for a single dielectric and a metal, and they have shown that the periodic metallic stopgap is associated with a decreasing absorption with the same periodicity [5]. The transmittance in Fig. 4 shows the characteristic behavior in Ref. [2], and it is possible to pinpoint on the  $\kappa_R$  behavior the origin of such a transmittance curves.

### 3. Conclusions

It is quite difficult to pinpoint the source of the behavior of the transmittance curves for thin metal MDPC [2] by themselves. To explain it, we have used the dispersion relation to complement the transmittance information. We have shown that is possible to relate the band structure with the transfer matrix method for 1D MDPC; what is directly related for ultrathin layers becomes complementary for thin layers. We have realized that the substratum as a dielectric or a DPC creates different decays, yet to be explained.

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